Walras law

Def The excess demand function of a pure-exchange economy (u, e) with demand functions $x_h(p)$ is

$$z(p) = \sum_h \left(x_h(p) - e_h \right),$$

$>0 \Rightarrow$ shortage

$<0 \Rightarrow$ glut

where $z: \mathbb{R}^N_{++} \to \mathbb{R}^N$.

Note: $(x^*(p^*), p^*)$ is an equilibrium $\Leftrightarrow z(p^*) = 0$ ← vector of 0s.

Theorem (Walras' Law) Consider a pure exchange economy (u, e) with strictly increasing utility functions and let z be its excess demand function. Then:

① $\underbrace{p \cdot z(p)}_{\sum_n p_n \cdot z_n(p)} = 0$ for all $\underbrace{p \in \mathbb{R}^N_{++}}$ all p, not just equilibrium prices!

② If $N-1$ markets clear at prices p, then all markets clear.

③ For every price vector p, the market fails to clear if and only if

ⓐ there is excess demand in one market, i.e. $z_n(p) > 0$ for some n, and

ⓑ there is excess supply in another market, i.e. $z_m(p) < 0$ for some m.

Proof

① Since each household has strictly increasing utility, households exhaust their budgets, i.e. $p \cdot x_h(p) = p \cdot e_h$.

Summing up gives

$$\sum_{h \in H} p \cdot x_h(p) = \sum_{h \in H} p \cdot e_h$$

$$\Leftrightarrow \sum_{h \in H} p \cdot (x_h(p) - e_h) = 0$$

$$\Leftrightarrow p \cdot \sum_{h \in H} (x_h(p) - e_h) = 0$$

$$\Leftrightarrow p \cdot z(p) = 0.$$

② WLOG, assume $z_n(p) = 0$ for $n = 1, 2, \dots, N-1$.

Adding up, and weighting by prices gives:

$$\sum_{n=1}^{N-1} p_n \underbrace{z_n(p)}_{=0} = 0$$

Subtracting this from ① gives

$$p_N z_N(p) = 0$$

$$\Leftrightarrow z_N(p) = 0.$$

(iii) Fix any price vector p.

excess supply/demand $\Rightarrow$ markets don't clear.

trivial (part of def)

Suppose markets don't clear, i.e. $z(p) \neq 0$. For the sake of contradiction, suppose there is no excess supply in any market, i.e. $z_n(p) \geq 0$ for n.

Then $p \cdot z(p) > 0$.

some are $=0$, some are >0.

This contradicts ①.