utility functions

Def Consider $x, y \in \mathbb{R}^N_+$. If a consumer <u>weakly prefers</u> x to y, we write $x \succsim y$. We say the consumer <u>strictly prefers</u> x to y, written $x \succ y$ if $x \succsim y$ but $y \not\succsim x$. We say the consumer is <u>indifferent</u> between x and y, written $x \sim y$ if $x \succsim y$ and $y \succsim x$.

Def A utility function $u: \mathbb{R}^N_+ \to \mathbb{R}$ <u>represents</u> the preferences $\succsim$ if for all $x, y \in \mathbb{R}^N_+$,

$$u(x) \geq u(y) \iff x \succsim y.$$

Properties:

* complete: either $x \succsim y$ or $y \succsim x$
* reflexive: $x \succsim x$
* transitive: if $x \succsim y$ and $y \succsim z$ then $x \succsim z$.
* continuous(*): if $x_n \to x^*$ and $x_n \succsim y$ for all n, then $x^* \succsim y$.

Theorem(*) Consider a preference relation $\succsim$. There exists a continuous utility function $u: \mathbb{R}^N_+ \to \mathbb{R}$ that represents $\succsim$ iff $\succsim$ is complete, reflexive, transitive and continuous.

Theorem Consider a preference relation $\succsim$, and any strictly increasing function $f: \mathbb{R} \to \mathbb{R}$. If u represents $\succsim$ then $v(x) = f(u(x))$ also represents $\succsim$.

Another property:

* <u>convexity</u>: all upper contour sets are convex sets. If $x \sim y$, then $tx + (1-t)y \succsim x \sim y$, where $t \in [0,1]$.

<u>Warning</u>: $u(c, \ell) = (c\ell)^{\frac{1}{4}}$. u is concave. $v(c, \ell) = c\ell$ represents the same preferences as u, but v is <u>not</u> concave. More info Afriat's theorem.

utility maximisation

$$\underbrace{v(\overset{\text{prices}}{p}, \overset{\text{wealth}}{m})}_{\text{indirect utility function}} = \max_{\substack{x \in \mathbb{R}^N_+ \\ \text{s.t. } p \cdot x \leq m}} u(x) = u(\underbrace{x(p,m)}_{\substack{\text{demand function} \\ \text{(optimal policy)}}})$$

another formulation:

$$v^*(p, \underset{\text{endowment}}{e}) = \max_{\substack{x \in \mathbb{R}^N_+ \\ \text{s.t. } p \cdot x \leq p \cdot e}} u(x) = u(x^*(p,e)).$$

FOC x_i: $$\left[\frac{\partial u(x)}{\partial x_i} - \lambda p_i\right]_{\substack{x = x(p,m) \\ \lambda = \lambda(p,m)}} = 0$$

$$\iff \frac{\left.\frac{\partial u(x)}{\partial x_i}\right|_{x = x(p,m)}}{p_i} = \lambda(p,m)$$

$$\Rightarrow \frac{\left.\frac{\partial u(x)}{\partial x_i}\right|_{x = x(p,m)}}{p_i} = \frac{\left.\frac{\partial u(x)}{\partial x_j}\right|_{x = x(p,m)}}{p_j}$$

$$\iff \left.\frac{\frac{\partial u(x)}{\partial x_i}}{\frac{\partial u(x)}{\partial x_j}}\right|_{x = x(p,m)} = \frac{p_i}{p_j}$$

slope of indiff curve (see: implicit function theorem)

slope of budget constraint