Slutsky equation

$$h(p, \bar{u}) = x(p, e(p, \bar{u}))$$

Theorem (Slutsky equation)

If the utility function u is smooth and the policies $x(p,m)$ and $h(p,\bar{u})$ are differentiable, then

$$\underbrace{\frac{\partial x_i(p,m)}{\partial p_j}}_{\text{net effect}} = \underbrace{\left[\frac{\partial h_i(p,\bar{u})}{\partial p_j}\right]_{\bar{u}=v(p,m)}}_{\text{subst effect}} + \underbrace{\underbrace{-x_j(p,m)}_{\text{lost wealth}} \underbrace{\frac{\partial x_i(p,m)}{\partial m}}_{\text{wealth effect on demand}}}_{\text{income effect}}$$

Proof: $h_i(p,\bar{u}) = x_i(p, e(p,\bar{u}))$.

Differentiate w.r.t. p_j:

$$\frac{\partial h_i(p,\bar{u})}{\partial p_j} = \left[\frac{\partial x_i(p,m)}{\partial p_j} + \frac{\partial x_i(p,m)}{\partial m}\frac{\partial e(p,\bar{u})}{\partial p_j}\right]_{m=e(p,\bar{u})}$$

$$= \left[\frac{\partial x_i(p,m)}{\partial p_j} + \frac{\partial x_i(p,m)}{\partial m} h_j(p,\bar{u})\right]_{m=e(p,\bar{u})}$$

This equation holds for all $(m,\bar{u})$ s.t. $m = e(p,\bar{u})$, $\Leftrightarrow$ $\bar{u} = v(p,m)$.

So we can rewrite:

$$\left.\frac{\partial h_i(p,\bar{u})}{\partial p_j}\right|_{\bar{u}=v(p,m)} = \frac{\partial x_i(p,m)}{\partial p_j} + \frac{\partial x_i(p,m)}{\partial m} x_j(p,m).$$

Rearranging gives the Slutsky equation.
$\square$

rice, beans, burgers

If $p_{rice} \uparrow$ then the cheapest to stay on the same indiff curve might be to $x_{burgers} \uparrow$, and $x_{rice} \downarrow$ and $x_{beans} \downarrow$. contradicts what I taught undergrads!