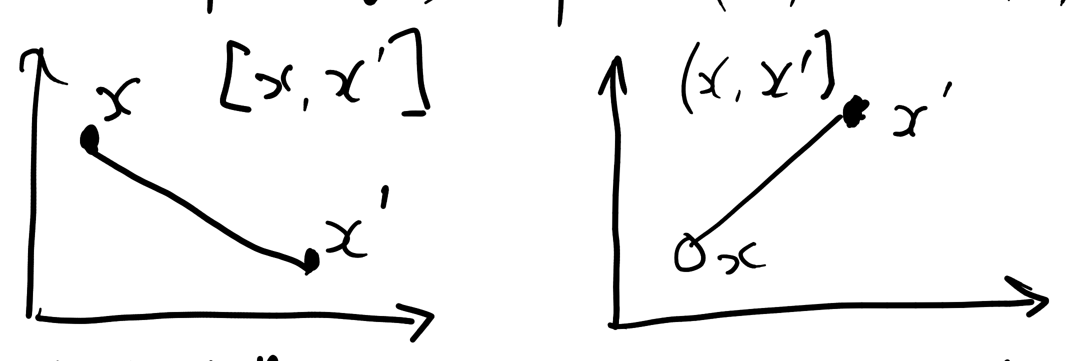
Def A <u>closed interval</u> between $x, x' \in \mathbb{R}^n$ is defined as

$$[x, x'] = \{\underbrace{ax + (1-a)x'}_{\text{a "mixture" or "convex combination" between } x \text{ and } x'} : a \in [0,1]\}$$

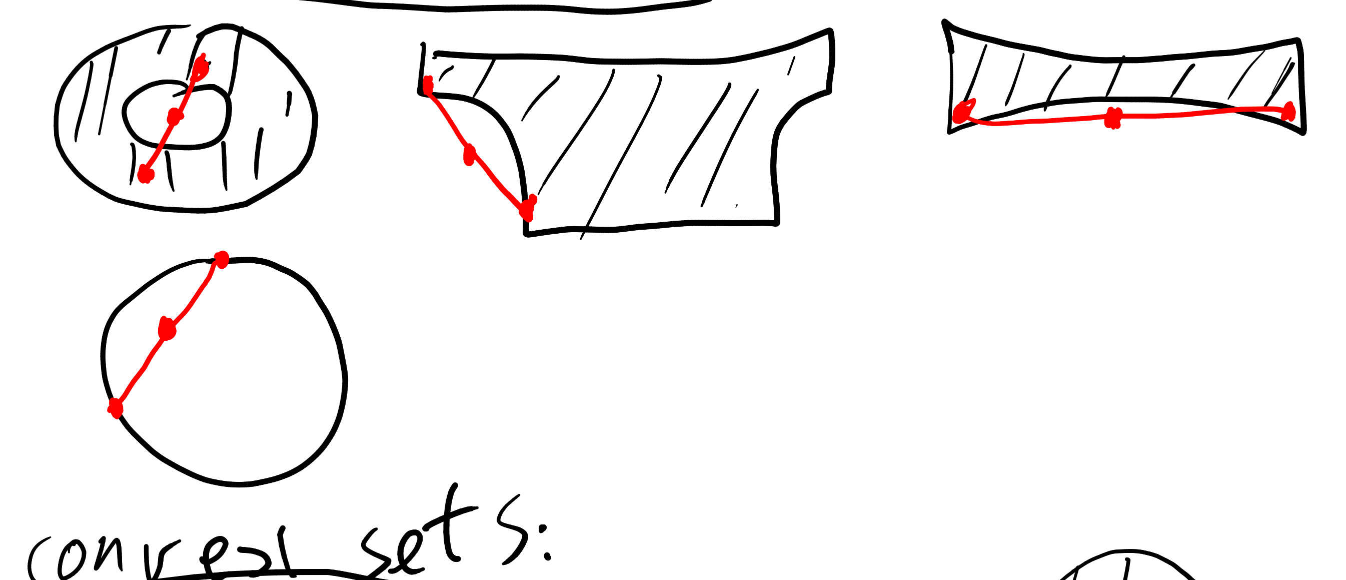
Similar definitions for (x, x') $(x, x']$, etc.



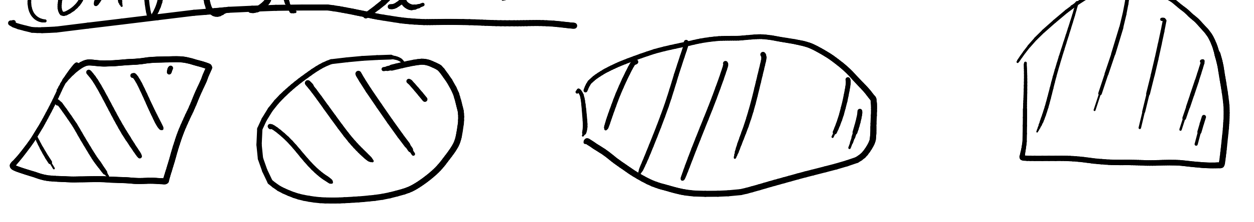
Def $X \subseteq \mathbb{R}^n$ is a <u>convex set</u> if for all $x, x' \in X$, the interval $[x, x']$ is a subset of X.

"all mixtures are on the menu"

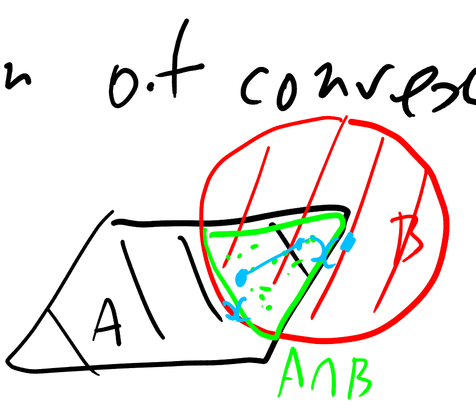
<u>non convex sets:</u>



<u>convex sets:</u>



Theorem The intersection of convex sets is convex.



Proof Suppose A and B are convex sets.

We want to prove that $A \cap R$ is a convex set. If $x, x' \in A \cap B$, then $x \in A$ and $x \in B$. Since A is convex, $[x, x'] \subseteq A$. Similarly, $[x, x'] \subseteq B$. Therefore $[x, x'] \subseteq A \cap B$. We conclude that $A \cap B$ is a convex set. □

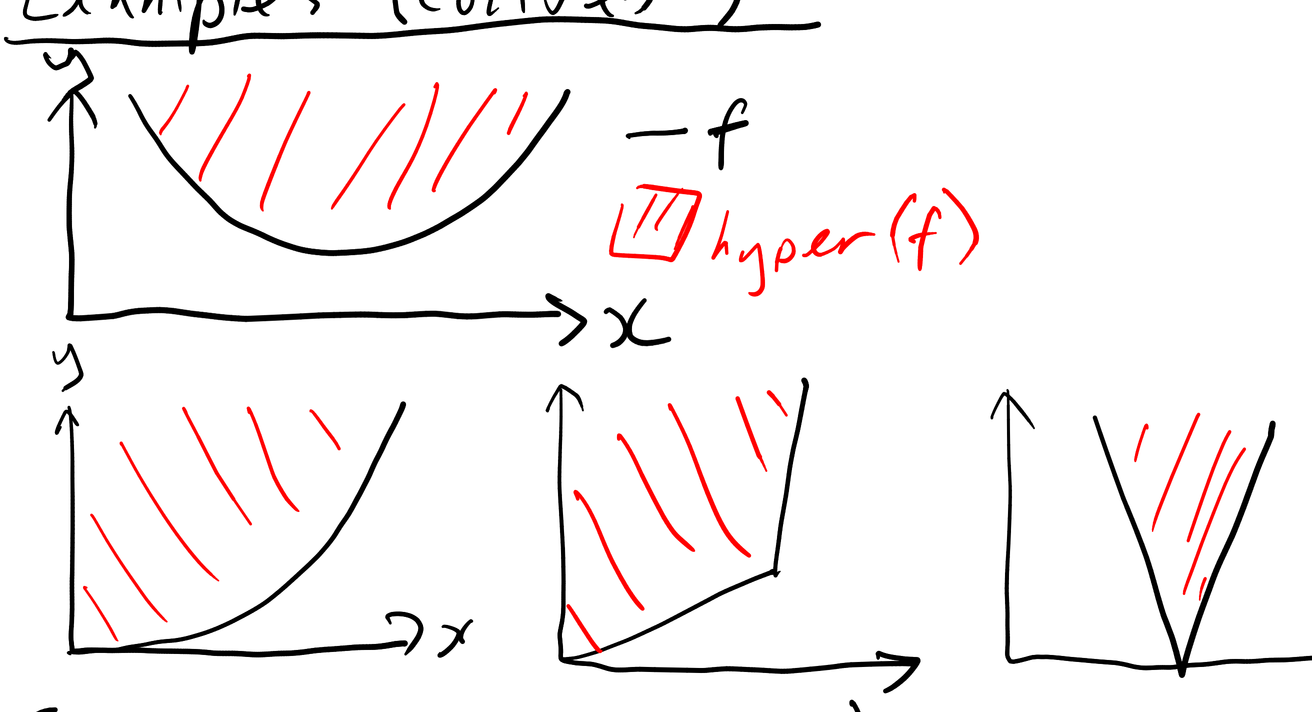
Def $f: X \to \mathbb{R}$ is a <u>convex function</u> if it's hypograph

$$\{(x, y) : x \in X,\ y \geq f(x)\}$$

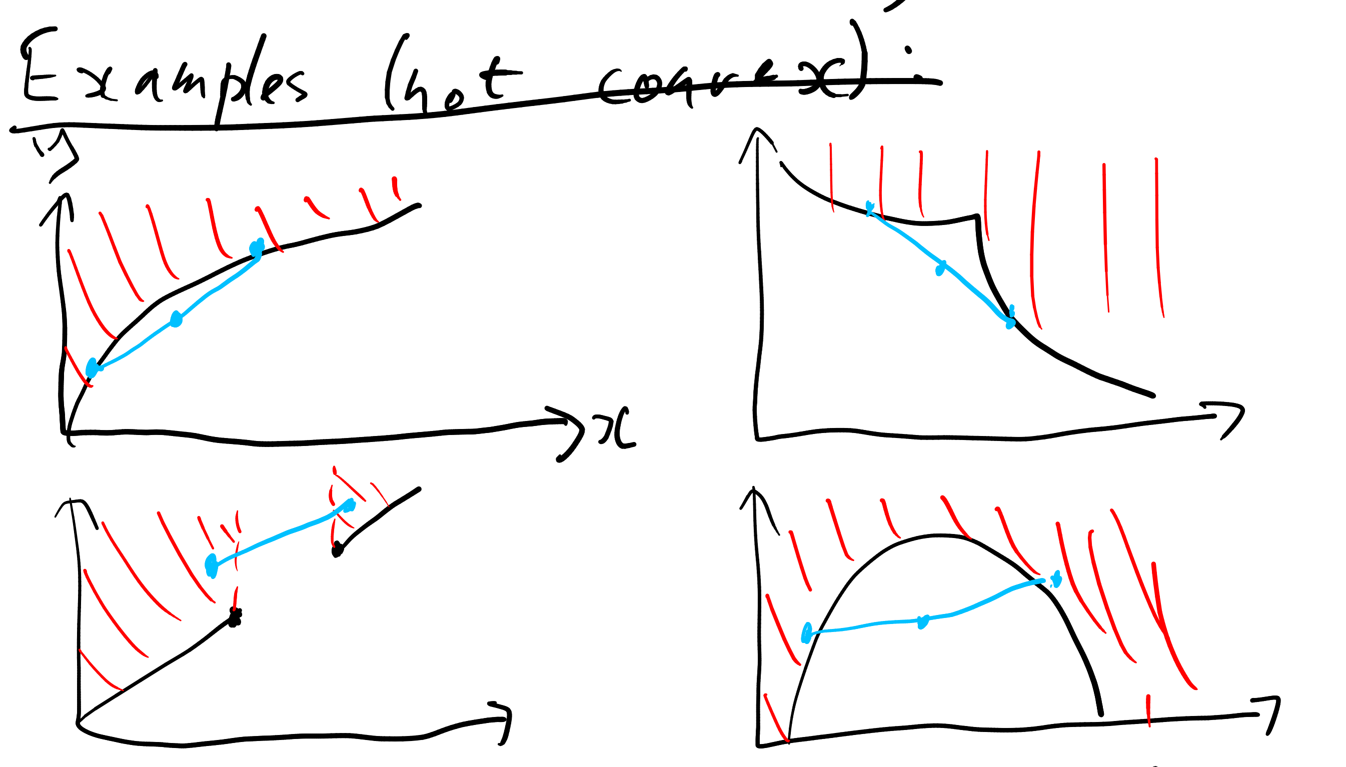
is a convex set.

Note: f can only be convex if its domain is a convex set.

<u>Examples (convex):</u>

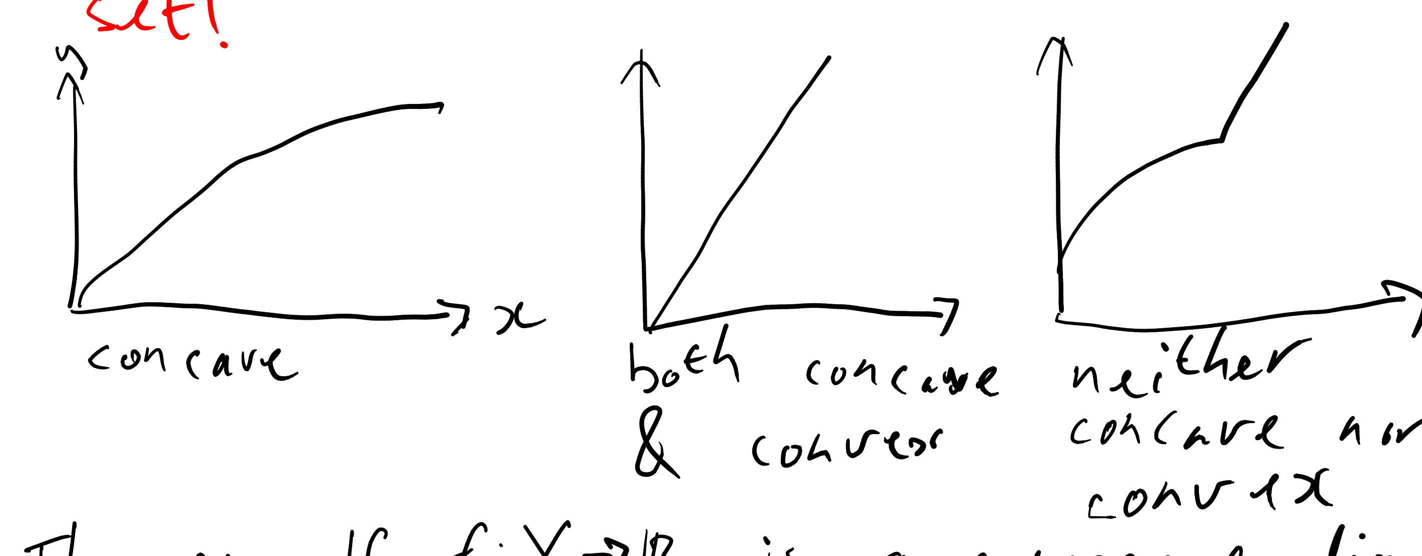


<u>Examples (not convex):</u>



Def $f: X \to \mathbb{R}$ is a <u>concave</u> function if $g(x) = -f(x)$ is a convex function. (Or, equivalently, if the <u>hypograph</u> of f is a convex set.)

Warning: no such thing as a concave set!



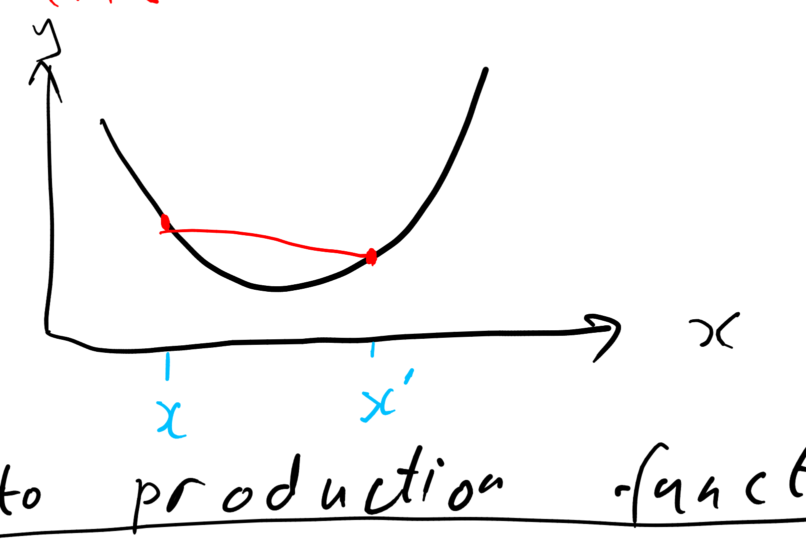
Theorem If $f: X \to \mathbb{R}$ is a convex function, and X is an open set, then f is a continuous.

Theorem Suppose $f: \mathbb{R} \to \mathbb{R}$ is differentiable. Then f is convex iff f' is weakly increasing.
if and only if

Theorem Suppose $f: \mathbb{R} \to \mathbb{R}$ is twice differentiable. Then f is convex iff $f''(x) \geq 0$ for all x.

Theorem A function $f: X \to \mathbb{R}$ is convex iff X is convex and for all $x, x' \in X$, and all $a \in [0,1]$,

$$\underbrace{\overset{\text{a "weight"}}{a} f(x) + (1-a) f(x')}_{\text{line}} \geq \underbrace{f(ax + (1-a)x')}_{\text{curve}}$$



<u>back to production functions</u>

* <u>f is concave:</u>

<u>claim</u> if f is smooth and concave, then it has weakly decreasing marinal productivity.

<u>claim</u> If f is concave and has the possibility of inaction, then f has weakly decreasing returns to scale.

Proof We want to prove that $f(tx) \leq t f(x)$ for $t > 1$.