N # goods
$N-1$ # inputs
1 output

$f : \mathbb{R}_+^{N-1} \to \mathbb{R}_+$, a production function.

(quantities of input) (quantity of output)

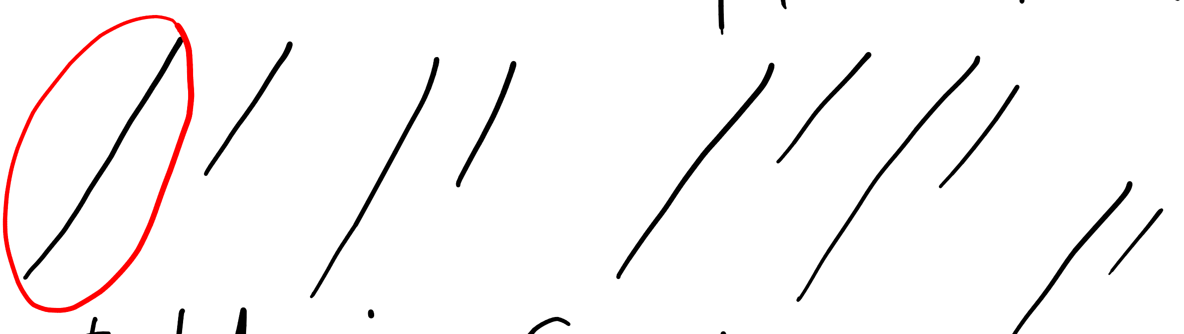
$f(k, l) = \sqrt{kl}$. But what if we don't the production function?

$x \in \mathbb{R}^{N-1}$ inputs eg: $x = (k, l)$
$y = f(x)$ output

Possible assumptions:

* possibility of inaction: $f(0) = 0$ ($(0, 0, \cdots, 0)$, $N-1$)

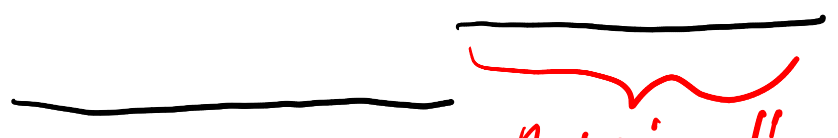
* free disposal: (monotonicity).
f is weakly monotonic increasing, i.e. if $x \geq x'$ ($x_n \geq x'_n$ for all n) then $f(x) \geq f(x')$. Stronger version:
f is strictly monotonic increasing
i.e. if $x > x'$ ($x \geq x'$, and $x_n > x'_n$ for some n) then $f(x) > f(x')$.



Bach Prelude in C major
maths: strictly monotonic increasing
music: mono = one tone = note. Not monotone! lots of notes

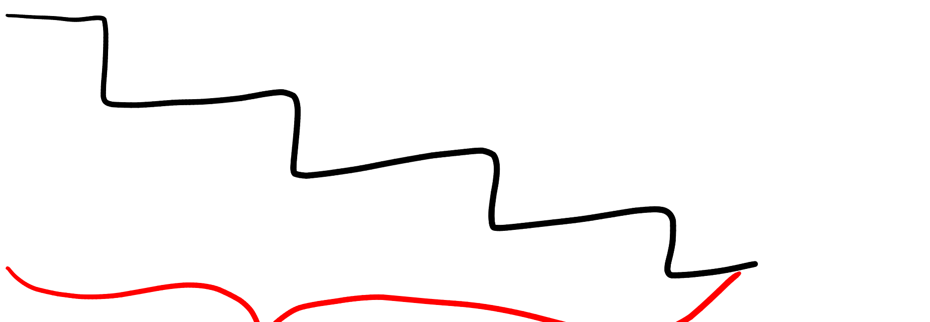
not monotone

One-note Samba



musically monotone
musically monotone
mathematically weakly monotone inc & dec!
mathematically weakly monotonic increasing.

Bach Jr



maths: weakly monotone decreasing

* smoothness: f is twice differentiable.
$\frac{\partial f(x)}{\partial x_i}$ is called the marginal product of x_i.

* decreasing marginal productivity.
$\frac{\partial f(x)}{\partial x_1}$ weakly monotone decreasing in x_1.

* weakly decreasing returns to scale:
for all $x \in \mathbb{R}_+^{N-1}$ and $t > 1$,
$f(tx) \leq t f(x)$.
(combined inputs) (combined output) (combined m.k.) (real mosque + original mosque kitchen)

* constant returns to scale:
for all $x \in \mathbb{R}_+^{N-1}$ and all $t > 0$,
$f(tx) = t f(x)$.

* weakly increasing returns to scale:
for all $x \in \mathbb{R}_+^{N-1}$ and all $t > 1$,
$f(tx) \geq t f(x)$.