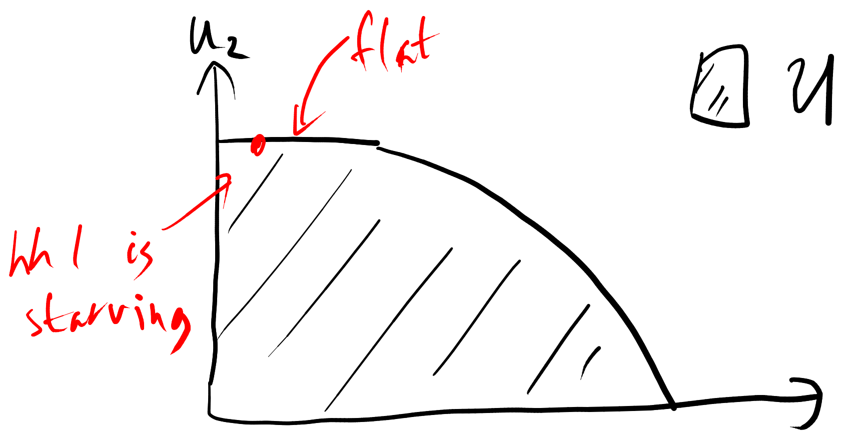
Efficient allocations

Def The utility possibility set for an economy is the set of possible utility vectors (one number per household). In a pure-exchange economy $(u_h, e_h)_{h\in H}$, this is

$$\mathcal{U} = \{(u_h(x_h))_{h\in H} : x \text{ is feasible}\}$$
$$= \Big\{(u_h(x_h))_{h\in H} : x_h \in \mathbb{R}^N_+ \text{ for all } h\in H \text{ and } \sum_{h\in H} x_h = \sum_{h\in H} e_h\Big\}.$$

Note: free disposal is convenient:

$$\sum_{h\in H} x_{hn} \le \sum_{h\in H} e_{hn} \quad \text{for all } n.$$



▨ $\mathcal{U}$

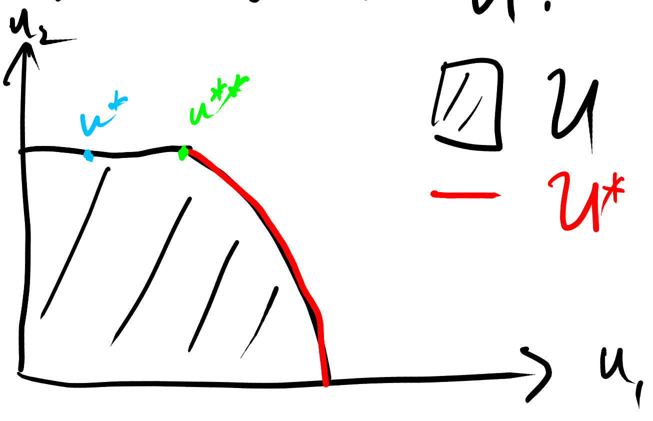
Def A vector of utilities $u \in \mathbb{R}^H$ Pareto dominates $u' \in \mathbb{R}^H$ if $u > u'$, i.e.

① no household is worse under u (vs u'), i.e. $u_h \ge u'_h$ for all $h\in H$, and

② there is at least one household $h\in H$ that is strictly better off, i.e. $u_h > u'_h$.

If an allocation is Pareto dominated by a feasible allocation, then it is (Pareto) inefficient. Otherwise, it is efficient.

Def The Pareto frontier of $\mathcal{U}$ is the set of efficient utility vectors. It is denoted $\mathcal{U}^*$.



▨ $\mathcal{U}$

— $\mathcal{U}^*$

Q: $u^* \in \mathcal{U}^*$?

A: No. $u^{**} > u^*$.

Def A social welfare function is a function $W : \mathbb{R}^H \to \mathbb{R}$.

\# households

Theorem Let $\mathcal{U} \subseteq \mathbb{R}^H$ be a utility possibility set, and $W : \mathbb{R}^H \to \mathbb{R}$ be a social welfare function. If

$$u \in \arg\max_{\hat{u}\in\mathcal{U}} W(\hat{u})$$ ⇐ u solves the social planner's problem

and if W is strictly increasing, then u is Pareto efficient, i.e. $u \in \mathcal{U}^*$.