

Statements

$x > y \Rightarrow x \geq y$ . (True!) *implies*

" $x$  is an even number"  $\leftarrow$  stronger vs "x is an integer" statement

"Suppose  $f: X \rightarrow \mathbb{R}$  is a differentiable function. If  $x^*$  maximises  $f$ , then  $f'(x^*) = 0$ ." **If A, then B implies C.**

Converse: If  $f'(x^*) = 0$ , then  $x^*$  maximises f. **(false!)**

contrapositive: If  $f'(x^*) \neq 0$ , then  $x^*$  does not maximise f. **not C implies not B.**

if and only if:  $f'(x^*) = 0$  iff  $x^*$  maximises f. **(false)**

Negation: of  $x = y$  is  $x \neq y$ .

## B5 Quantifiers

" $x$  is an even number" true or false?

" $\cos^2 x + \sin^2 x = 1$  for all  $x \in \mathbb{R}$ ."  
for every, for any

"there exists some  $x \in \mathbb{R}$  such that  $\sin x = 1$ ."  
there is some, there is, for some

(i) For all criminals c, there exists a punishment p such that the criminal c would be deterred from crime.

(ii) There exists a punishment p such that all criminals c would be deterred from crime

## B6 Theorems and Proofs

"Do you agree that the British Empire was a force for a good?"

"Prove that if the maximum of a set exists, then it equals its supremum."

◻ QED  $\leftarrow$  end of a proof  
↳  $\leftarrow$  end of a proof by contradiction