(40) (i) 2 × pizza, blintzes, skilled, unskilled

$\frac{1}{9}$ × education

markets

P_{pt} price of pizza in period t

P_{bt} " " blintzes " " "

w_{st} skilled wages in period t

w_{ut} unskilled " " "

P_{e1} tuition fees

Russian household

Assuming inelastic skilled labour supply $l^r_{st} = 1$.

x^r_{pt} pizza consumption

x^r_{bt} blintzes consumption

π^r_{rest}, π^r_{school} firm profits

utility $u(x^r_{p1}, x^r_{b1}) + u(x^r_{p2}, x^r_{b2})$

They solve:

$$\max_{x^r_{pt}, x^r_{bt}} u(x^r_{p1}, x^r_{b1}) + u(x^r_{p2}, x^r_{b2})$$

$$\text{s.t. } \sum_{t=1}^{2} \{P_{bt} x^r_{bt} + P_{pt} x^r_{pt}\}$$

$$\leq \sum_{t=1}^{2} w_{st} \cdot 1 + \pi^r_{rest} + \pi^r_{school}$$

Local problem Let H^e be the set of Edinburgh households. Then household $h \in H^e$ chooses

x^h_{pt} pizza consumption in period t

x^h_{bt} blintzes " " "

e^h education choice, 0 or 1 takes time

receive profits $\frac{\pi_{pizza}}{|H^e|}$.

Assume same utility as Russians.

They solve:

$$\max_{x^h_{pt}, x^h_{bt}, e \in \{0,1\}} \sum_{t=1}^{2} u(x^h_{pt}, x^h_{bt})$$

$$\text{s.t. } \sum_{t=1}^{2} \{P_{pt} x^h_{pt} + P_{bt} x^h_{bt}\} + P_e e$$

$$\leq w_{u1}(1 - \underbrace{l_e}_{\text{time spent studying}} e) + [e w_{s2} + (1-e) w_{u2}] \cdot 1$$

pizza restaurant: hires L^p_t unskilled workers (cutting corners!) in period t, and produces $P_t = f(L^p_t)$ pizzas. Its problem is

$$\pi_{pizza}(P_{p1}, P_{p2}; w_{u1}, w_{u2}) = \max_{L^p_1, L^p_2} \sum_{t=1}^{2} \{P_{pt} f(L^p_t) - w_{ut} L^p_t\}.$$

Russian restaurant hires L^b_t skilled workers in period t, and produces $B_t = g(L^b_t)$ blintzes. Profit function:

$$\pi_{blintzes}(P_{b1}, P_{b2}; w_{s1}, w_{s2}) = \max_{L^b_1, L^b_2} \sum_{t=1}^{2} \{P_{bt} g(L^b_t) - w_{st} L^b_t\}$$

Russian school hires L^s workers in period 1, and produces $E = h(L^s)$ school places. Profit function:

$$\pi_{school}(P_{e1}; w_{s1}) = \max_{L^s} P_{e1} h(L^s) - w_{s1} L^s.$$

Equilibrium prices $(P_{bt}, P_{pt}, P_e, w_{st}, w_{ut})$ and quantities $(x^r_{pt}, x^r_{bt}, x^h_{pt}, x^h_{bt}, e^h, L^p_t, P_t, L^b_t, B_t, E, L^s)$ form an equilibrium if each choice solves the relevant problem above, and all markets clear:

$$x^r_{p1} + \sum_{h \in H^e} x^h_{p1} = P_1$$

$$x^r_{p2} + \sum_{h \in H^e} x^h_{p2} = P_2$$

$$x^r_{b1} + \sum_{h \in H^e} x^h_{b1} = B_1$$

$$x^r_{b2} + \sum_{h \in H^e} x^r_{b2} = B_2$$

$$\underbrace{1 + 0}_{\text{supply of skilled labour in period 1}} = \underbrace{L^s + L^B_1}_{\text{demand of skilled labour in period 1}}$$

$$\underbrace{\sum_{h \in H^e} \{1 - e_h l_e\}}_{\text{unskilled } t=1 \text{ labour supply}} = \underbrace{L^p_1}_{\text{unskilled labour demand } t=1}$$

$$1 + \sum_{h \in H^e} e_h = L^B_2$$

$$\sum_{h \in H^e} (1 - e_h) = L^P_2$$

$$\sum_{h \in H^e} e_h = E$$

(ii) Since π^{school} is the upper envelope of linear ($\Rightarrow$ convex) functions (one function per L^s), we know π^{school} is a convex function.

By the envelope theorem

$$\frac{\partial \pi^{school}}{\partial w_{s1}} = \left[\frac{\partial}{\partial w_s}\{P_e h(L^s) - w_s L_s\}\right] \Big|_{L_s = L_s(P_e; w_{s1})}$$

$$= [-L_s]\Big|_{L_s(P_e; w_{s1})}$$

$$= -L_s(P_e; w_{s1}).$$

Since π^{school} is convex, the left side is increasing in w_{s1}.

So the right side is too.

So $L_s(P_e; w_{s1})$ is decreasing in w_{s1}.

(iv) $V(e)$, the value of education, is

$$V(e) = \max_{x^h_{pt}, x^h_{bt}} \sum_{t=1}^{2} u(x^h_{pt}, x^h_{bt})$$

e moved

$$\text{s.t. } \sum_{t=1}^{2} \{P_{pt} x^h_{pt} + P_{bt} x^h_{bt}\} + P_e e$$

$$\leq \text{same as before}$$

Bellman eq: $\max_{e \in \{0,1\}} V(e)$. = indirect utility function

(v) Want to prove, in every eq, all local households consume the same.

If some households study, and others don't, then $V(0) = V(1)$.

(otherwise, everyone would switch!)

So the wealth of all locals is the same, i.e.:

$$w_{u1} + w_{u2} + \frac{\pi^{pizza}}{|H^e|}$$

$$= w_{u1}(1 - l_e) - P_e + w_{u2} + \frac{\pi^{pizza}}{|H^e|}.$$

Both $e=0$ and $e=1$ households solve the same problem (RHS $V(\cdot)$).

This problem has a unique solution if the utility function is strictly concave.

So all locals consume the same food.