Def (X, d) is a <u>metric space</u> if X is a set,

set of points distance metric

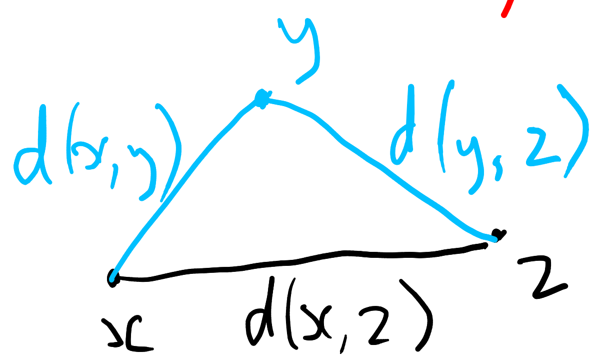
and $d: X \times X \to \mathbb{R}_+$ satisfies

(i) $d(x,y) = 0$ iff $x = y$,

(ii) $d(x,y) = d(y,x)$ for all $x, y \in X$, and

(iii) $d(x,z) \leq d(x,y) + d(y,z)$ for $x, y, z \in X$

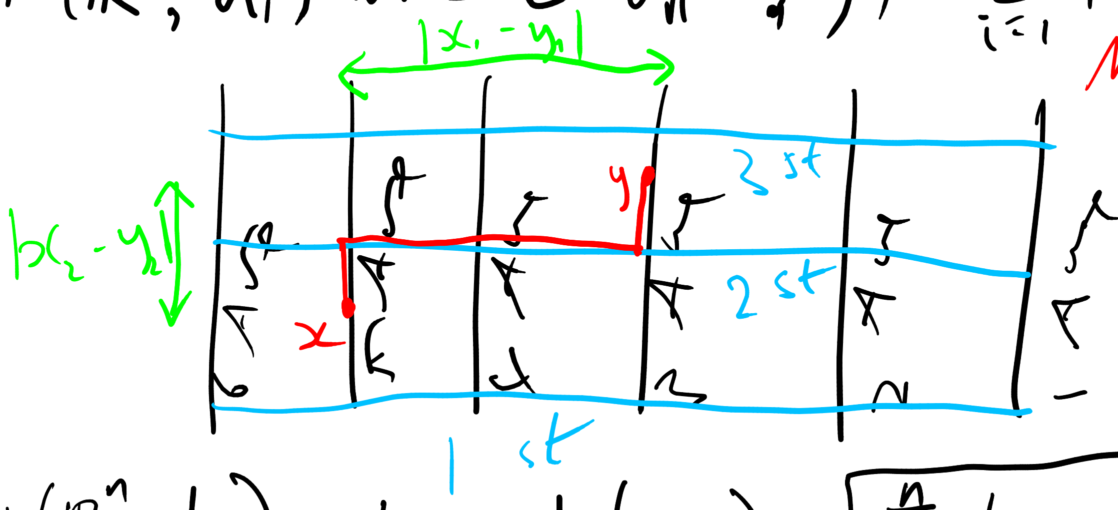
triangle inequality



Examples:

* $(\mathbb{R}^n, d_1)$ where $d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|$.

Manhattan metric



* $(\mathbb{R}^n, d_2)$ where $d_2(x,y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$

Euclidean metric (Pythagoras metric?)

* $(\mathbb{R}^n, d_\infty)$ where $d_\infty(x,y) = \max_i |x_i - y_i|$,

$= \max\{|x_1 - y_1|, |x_2 - y_2|, \ldots, |x_n - y_n|\}$.

* (X, d) where X is any set, and $d(x,y) = 1$ if $x \neq y$, and 0 if $x = y$.

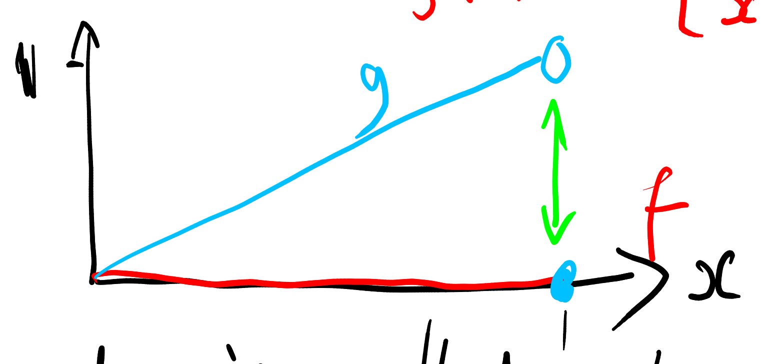
Discrete metric

* $X = \{f : [0,1] \to [0,1]\}$ and

$d_\infty(f,g) = \cancel{\max}\ \sup_{x \in [0,1]} |f(x) - g(x)|$.

Problem: $f(x) = 0$,

$g(x) = \begin{cases} 0 & \text{if } x = 1 \\ x & \text{if } x \in [0,1). \end{cases}$



$\sup [0,1) = 1$.

d_∞ is called the supremum metric (or "sup" or "uniform" metric).

<u>Not metric spaces:</u>

* $(\mathbb{R}^n, d)$ where $d(x,y) = \min_i |x_i - y_i|$?

Problem: $x = (0,1)$ and $y = (0,0)$.

This violates property i:

$d(x,y) = 0$ but $x \neq y$.

* $(\mathbb{R}^n, d)$ where $d(x,y) = 0$. Same problem.