Supply curve shape:

$$y(p;w) = \frac{\partial \pi(p;w)}{\partial p}$$

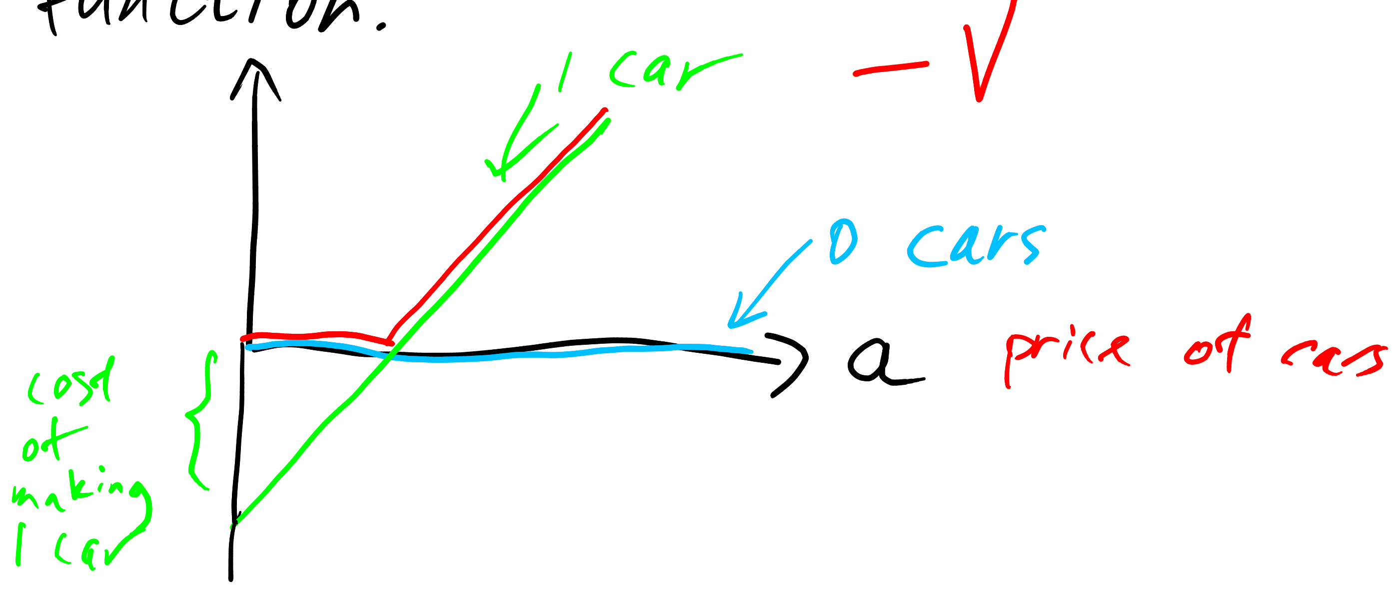
$$\frac{\partial y(p;w)}{\partial p} = \frac{\partial^2 \pi(p,w)}{\partial p^2} \quad >0?$$

Similarly:

$$x_n(p,w) = -\frac{\partial \pi(p;w)}{\partial w_n}$$

$$\frac{\partial x_n(p,w)}{\partial w_n} = -\frac{\partial^2 \pi(p,w)}{\partial w_n^2} \quad <0?$$

Theorem Suppose $V(a) = \max_b v(a,b)$. If each $v(\cdot, b)$ is a convex function, then V is a convex function.



Proof We want to prove that for every a (poor) and a' (rich) and every $t \in [0,1]$

$$\underbrace{tV(a) + (1-t)V(a')}_{\text{line}} \geq \underbrace{V(ta+(1-t)a')}_{\text{curve}}$$

$$tV(a) + (1-t)V(a')$$

$$= t\,v(a, b(a)) + (1-t)\,v(a', b(a'))$$

2 eggs + 18 bananas; caviar

$$\geq t\,v(a, \underbrace{b(ta+(1-t)a')}_{\text{burger}}) + (1-t)\,v(a', b(a'))$$

$$\geq t\,v(a, b(ta+(1-t)a')) + (1-t)\,v(a', \underbrace{b(ta+(1-t)a')}_{\text{burger}})$$

$$\geq v(\underbrace{ta+(1-t)a'}_{\text{middle income}}, \underbrace{b(ta+(1-t)a')}_{\text{burger}})$$

$$= V(ta+(1-t)a'). \qquad \square$$

Theorem 2.3 For every production function f, if the corresponding profit function π is smooth, then $\frac{\partial y(p;w)}{\partial p} \geq 0$ and $\frac{\partial x_n(p;w)}{\partial w_n} \leq 0$.

Proof First we show that π is a convex function. Recall

$$\pi(p;w) = \max_x pf(x) - w \cdot x.$$

Let $v(p,w;x) = pf(x) - w \cdot x$ $= (p,w)\cdot(f(x), -x)$, so

$$\pi(p;w) = \max_x v(p,w;x).$$

Now v is linear (and hence convex) in (p,w). So the previous theorem establishes that π is a convex function.

Second, since π is convex, it is also convex as a function of p only (holding w) fixed. Since we assumed that π is smooth, we deduce

$$\frac{\partial^2 \pi(p;w)}{\partial p^2} \geq 0.$$

But we know $\frac{\partial y(p;w)}{\partial p} = \frac{\partial^2 \pi(p;w)}{\partial p^2}$.

We conclude that $\frac{\partial y(p;w)}{\partial p} \geq 0$.

The proof for factor demands is similar. $\square$

(i)
- d — wholesale food quantity
- ϕ — wholesale food price
- ℓ — labour
- w — wages
- p — retail price

$f(\ell, d)$ retail food output

profit function:

$$\pi(p;\phi,w) = \max_{\ell,d} pf(\ell,d) - w\ell - \phi d.$$

(ii) π is convex:

The firm's objective is linear in (p,ϕ,w), so π is the upper envelope of linear functions (one function for each (ℓ,d)). Therefore, π is convex.

(iii) $\frac{\partial d(p;\phi,w)}{\partial \phi} \leq 0$:

By the envelope theorem:

$$\frac{\partial \pi(p;\phi,w)}{\partial \phi} = \left[\frac{\partial}{\partial \phi}\{pf(d,\ell) - w\ell - \phi d\}\right]_{\substack{\ell = \ell(p;\phi,w) \\ d = d(p;\phi,w)}}$$

$$= [-d]_{\substack{\ell = \ell(p;\phi,w) \\ d = d(p;\phi,w)}}$$

$$= -d(p;\phi,w).$$

Rearrange: $d(p;\phi,w) = -\frac{\partial \pi(p;\phi,w)}{\partial \phi}$

So

$$\frac{\partial d(p,\phi,w)}{\partial \phi} = -\frac{\partial^2 \pi(p;\phi,w)}{\partial \phi^2}.$$

The right side is ≤ 0 since π is convex. We conclude the left side is ≤ 0. $\square$