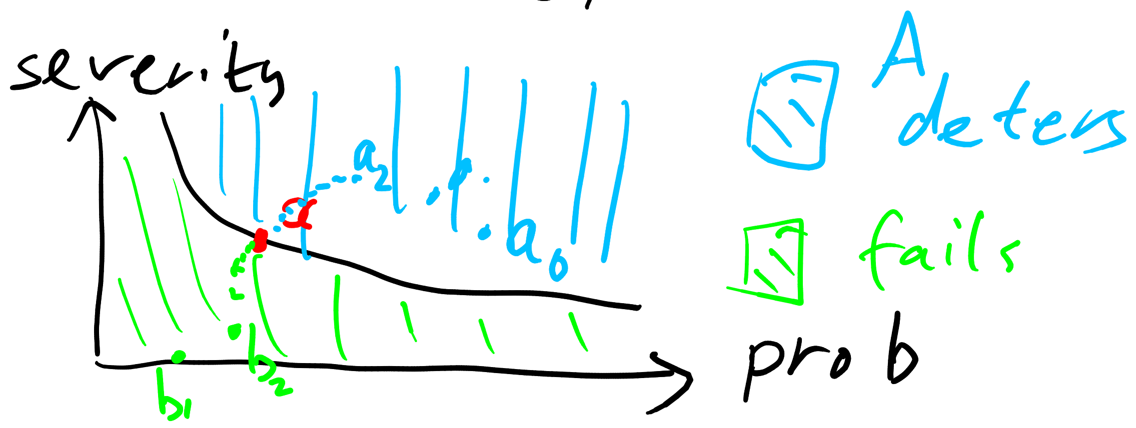
Def Let A be any subset of a metric space (X, d). A point $x \in X$ is a <u>boundary point</u> of A if

(i) there exists a "inside" sequence $a_n \in A$ such that $a_n \to x$, and

(ii) there exists a "outside" sequence $b_n \in X \setminus A$ such that $b_n \to x$.

The set of boundary points of A is called the <u>boundary</u> of A, denoted ∂A.



Note one of these sequences could be the trivial (boring) sequence $x, x, x, \dots$

More examples:

* $\partial [0,1]$ in $(\mathbb{R}, d_2)$ is $\{0,1\}$.

  e.g. $0 \in \partial [0,1]$ because
  $\left.\begin{array}{l} a_n = 0 \\ b_n = -\frac{1}{n} \end{array}\right\} \to 0.$

  $\frac{1}{2} \notin \partial [0,1]$ because there is no sequence $b_n \notin A$ with $b_n \to x$.

* $\partial (0,1)$ in $(\mathbb{R}, d_2)$ is $\{0,1\}$.

  e.g. $0 \in \partial (0,1)$ because $a_n = \frac{1}{n} \to 0$, same b_n.

* $\partial [0,1]$ in $([0,1], d_2)$ is $\emptyset$.

  $0 \notin \partial [0,1]$ because there is no sequence $b_n \in X \setminus A$ s.t. $b_n \to x$.

* $\partial (0,1)$ in $([0,1], d_2)$ is $\{0,1\}$.

  $0 \in \partial (0,1)$ because $a_n = \frac{1}{n} \to x$ and $b_n = 0 \to x$.

* $\partial [0,1]$ in $(\mathbb{R}_+, d_2)$ is $\{1\}$.