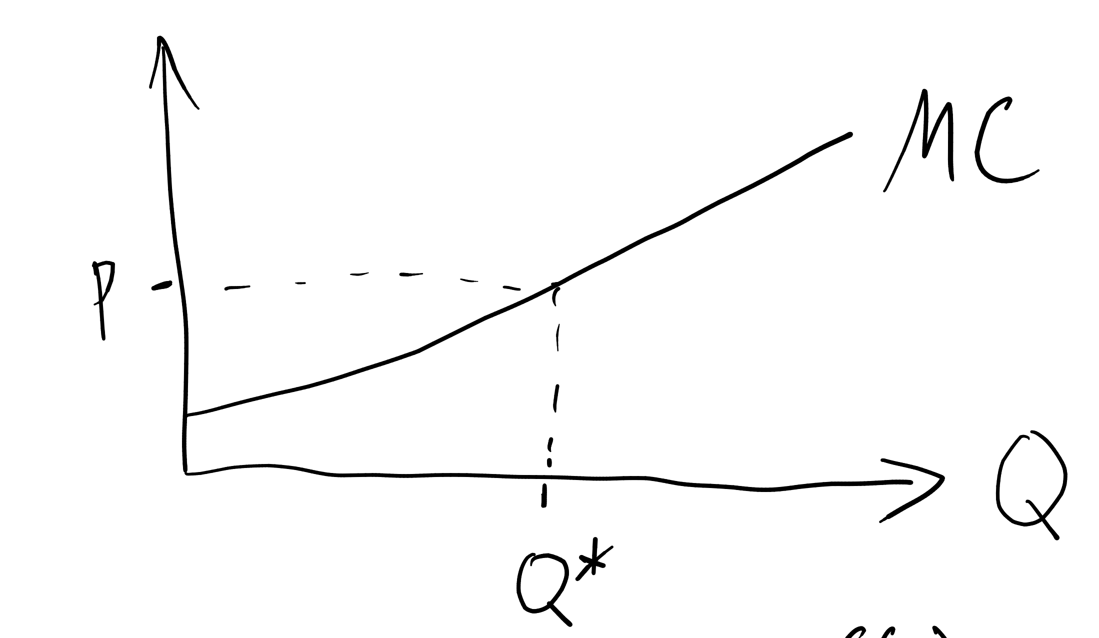

$$\pi(p;w) = \max_{x \in \mathbb{R}^{N-1}_+} pf(x) - w \cdot x.$$

$$c(y;w) = \min_{x \in \mathbb{R}^{N-1}_+} w \cdot x \quad \text{s.t. } f(x) \geq y.$$

$$\boxed{\pi(p;w) = \max_{y \in \mathbb{R}_+} py - c(y;w).}$$

Bellman equation
- two value functions (π & c)
- connected via an equation
  * one value function on LHS
  * max/min on RHS
  * other value function on RHS

Bellman (1957) — dynamic programming.
* optimal rocket trajectory
* genetics ACGTT TAACC ...
  AG-TTTCGACC ...
* network routing
* sorting - alphabetical order
* consumption/savings trade-off

Principle of Optimality check that the Bellman equation holds.

$$\max_y py - c(y;w)$$
$$= \max_y py - \left[\min_{x \in \mathbb{R}^{N-1}_+} w \cdot x \text{ s.t. } f(x) \geq y\right]$$
$$= \max_y py + \left[\max_x -w \cdot x \text{ s.t. } f(x) \geq y\right]$$
$$= \max_y \left[\max_x py - w \cdot x \text{ s.t. } f(x) \geq y\right]$$
$$= \max_{x,y} py - w \cdot x \text{ s.t. } f(x) \geq y$$
$$= \max_{x,y} py - w \cdot x \text{ s.t. } f(x) = y \quad \text{(free disposal)}$$
$$= \max_{x,y} pf(x) - w \cdot x \quad f(x) = y$$
$$= \max_x pf(x) - w \cdot x$$
$$= \pi(p,w). \quad \square$$

Theorem If $y(p;w)$ is an optimal supply policy, then for all prices (p,w),

$$p = \underbrace{\left.\frac{\partial c(y;w)}{\partial y}\right|_{y=y(p;w)}}_{\text{marginal cost}}$$

Proof By the principle of optimality,

$$\pi(p;w) = \max_y py - c(y;w).$$

Then take the FOC with respect to y. $\square$

Re-apply the envelope to the Bellman equation:

$$\frac{\partial \pi(p;w)}{\partial p} = \left[\frac{\partial}{\partial p}\{py - c(y;w)\}\right]_{y=y(p,w)}$$
$$= [y]_{y=y(p,w)}$$
$$= y(p,w). \quad \text{same as before}$$

$$\frac{\partial \pi(p;w)}{\partial w_n} = \left[\frac{\partial}{\partial w_n}\{py - c(y;w)\}\right]_{y=y(p,w)}$$
$$= \left[-\frac{\partial c(y;w)}{\partial w_n}\right]_{y=y(p,w)}$$