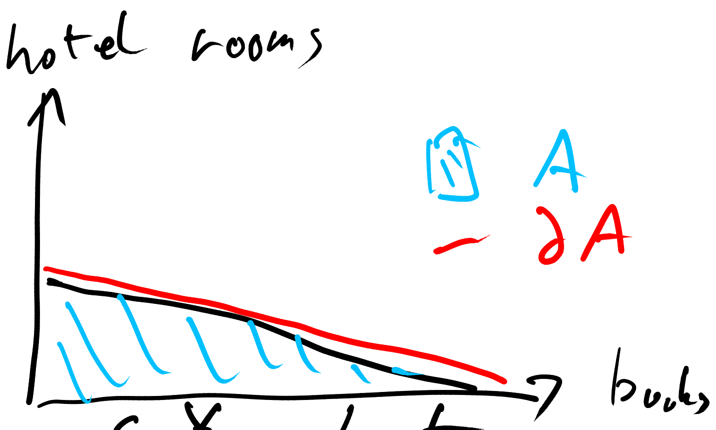
C.36 Suppose $p \in \mathbb{R}^N_{++}$.
Let $A = \{x \in \mathbb{R}^N_+ : p \cdot x \le m\}$.
Prove A is a closed set inside $(\mathbb{R}^n_+, d_2)$.

Proof Let $f(x) = p \cdot x = \sum_{n=1}^{N} p_n x_n$.
f is continuous.
Note that $A = \underbrace{f^{-1}([0,m])}_{\text{spending} \le m}$.

what bundles cost this much

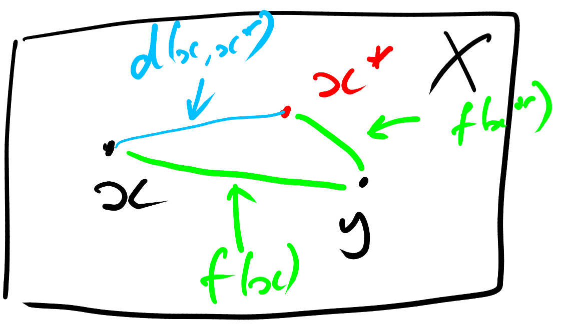
Since $[0,m]$ is closed and f is continuous, $f^{-1}([0,m])$ is closed. So A is closed.

$f : \underset{d_2}{\mathbb{R}^N_+} \to \mathbb{R}$



C.41 Fix any $y \in X$. Let $f(x) = d(x,y)$. Prove that f is continuous, where $f : X \to \mathbb{R}$.

We will prove
$|f(x) - f(x^*)| \le d(x, x^*)$



Suppose $x_n \to x^*$. We want to prove $f(x_n) \to f(x^*)$.

$d_2(f(x_n), f(x^*)) = |f(x_n) - f(x^*)| \le d(x_n, x^*) \to 0$

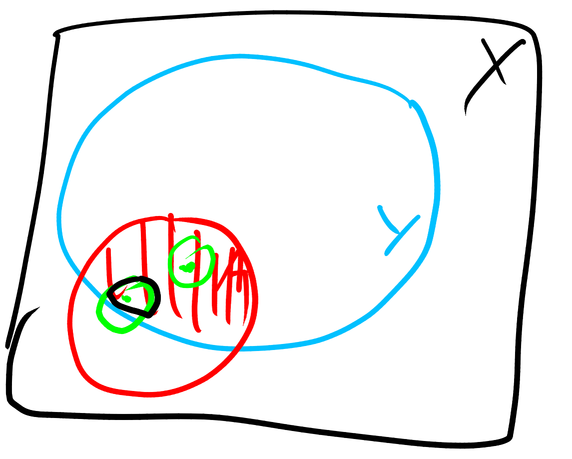
So $|f(x_n) - f(x^*)| \to 0$.

$\iff d_2(f(x_n), f(x^*)) \to 0$

$\iff f(x_n) \to f(x^*)$.

Claim Suppose A is an open set inside (X,d). Let $Y \subseteq X$. Then $A \cap Y$ is an open set inside (Y,d).

Proof Pick any point $a \in A \cap Y$. Since A is open inside (X,d), there exists some ball $B^X_r(a) \subseteq A$.



Then $B^Y_r(a) = \{y \in Y : d(a,y) < r\} \subseteq A \cap Y$.
So $A \cap Y$ is an open set in (Y,d).

4 (v) is not part of the course — my mistake!

4 (iv)