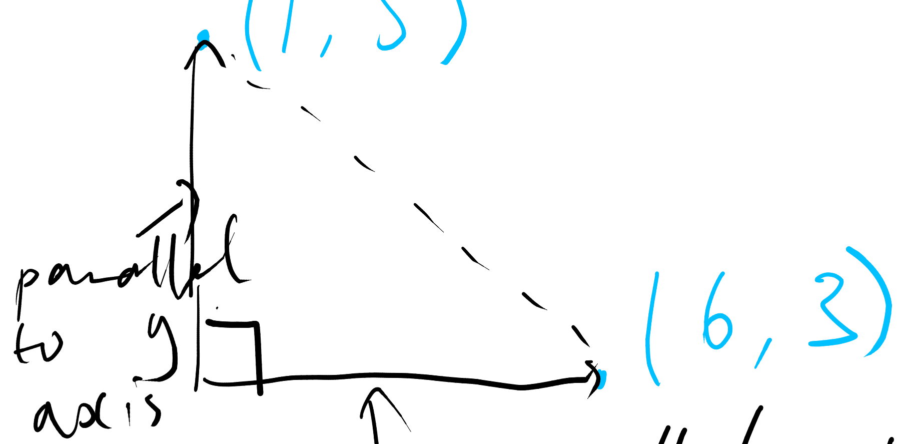
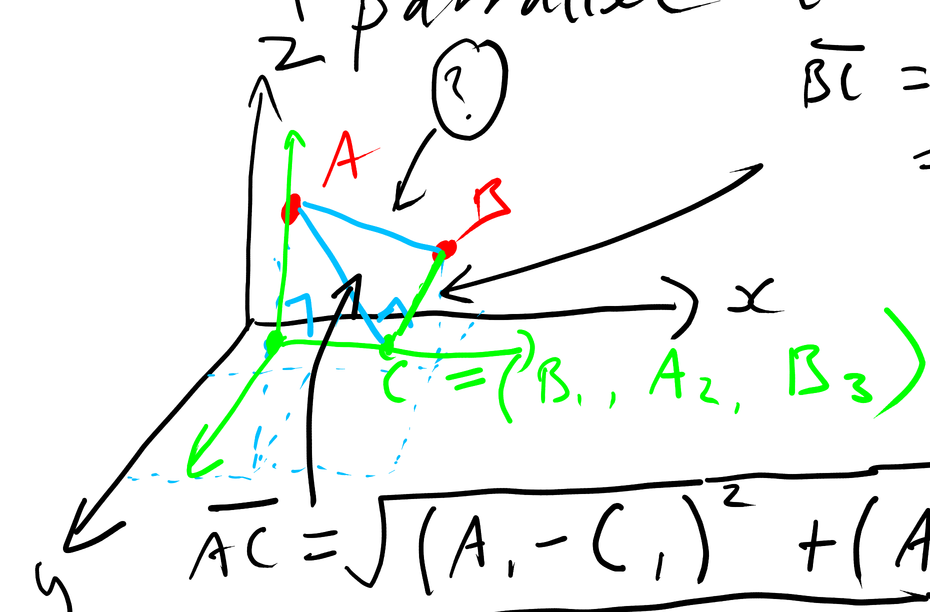
Pythagoras:



parrallel to x axis

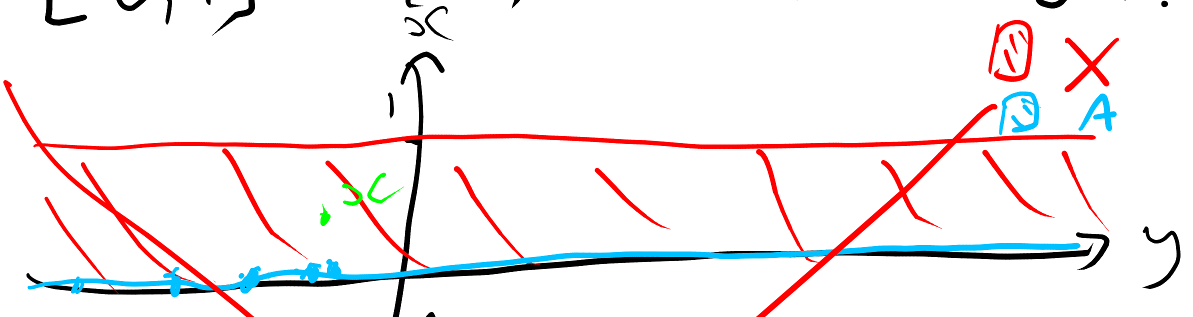
$\overline{BC} = |B_2 - C_2|$
$= |B_2 - A_2|$



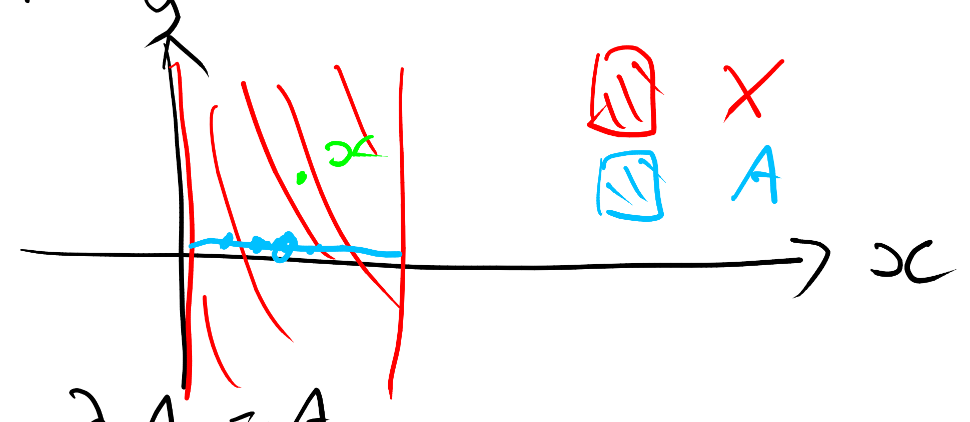
$\overline{AC} = \sqrt{(A_1 - C_1)^2 + (A_3 - C_3)^2}$
$= \sqrt{(A_1 - B_1)^2 + (A_3 - B_3)^2}$

$\overline{AB} = \sqrt{\overline{AC}^2 + \overline{BC}^2}$
$= \sqrt{\{(A_1 - B_1)^2 + (A_3 - B_3)^2\} + \{(B_2 - A_2)^2\}}$

24 B (i) $X = [0,1] \times \mathbb{R}$ and d_2.
$A = [0,1] \times \{0\}$. What is ∂A?



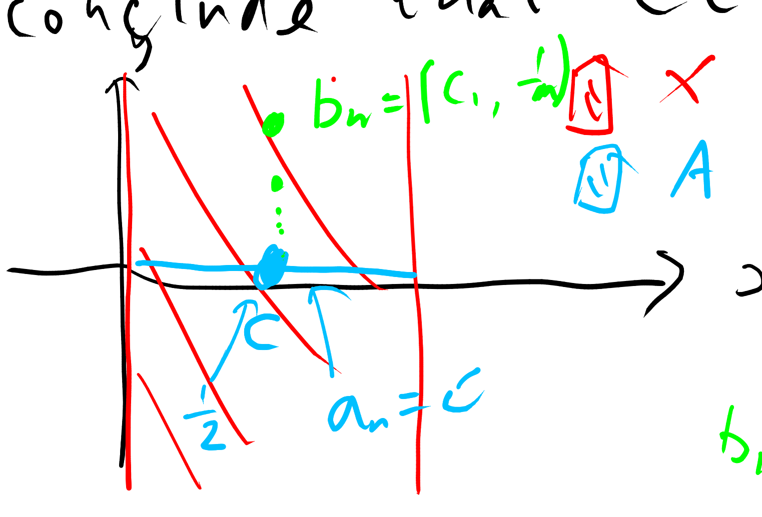
~~Claim: $\partial A = A$.~~
~~Proof ① $\partial A \subseteq A$: Suppose $x \in \partial A$.~~
~~Then there exists a sequence~~
~~$a_n \in A$ s.t. $a_n \to x$.~~
~~So $d_2(a_n, x) \to 0$.~~
~~So $|(a_n)_2 - x_2| \to 0$~~



Claim $\partial A = A$.
Proof ① $\partial A \subseteq A$: Pick any point $x \in \partial A$. Then there exists some $a_n \in A$ s.t. $a_n \to x$.
So $d_2(a_n, x) \to 0$.
So $\sqrt{\underbrace{\{(a_n)_1 - x_1\}^2}_{\geq 0} + \underbrace{\{(a_n)_2 - x_2\}^2}_{\geq 0}} \to 0$

So $(a_n)_1 - x_1 \to 0$
and $\underbrace{(a_n)_2}_{=0} - x_2 \to 0$.
So $0 - x_2 \to 0 \Rightarrow x_2 = 0$.
So $x \in A$.

② $A \subseteq \partial A$: Pick any $c \in A$.
To prove $c \in \partial A$, we need an "inside" sequence and an "outside" sequence.
* Let $a_n = c$. So $a_n \to c$.
* Let $b_n = (c, \frac{1}{n})$. So $b_n \notin A$ and $b_n \to c$.
($\frac{1}{n} \to 0$)
We conclude that $c \in \partial A$. □



$b_1 = (\frac{1}{2}, 1)$
$b_2 = (\frac{1}{2}, \frac{1}{2})$
$b_3 = (\frac{1}{2}, \frac{1}{3})$
$b_{100} = (\frac{1}{2}, \frac{1}{100})$