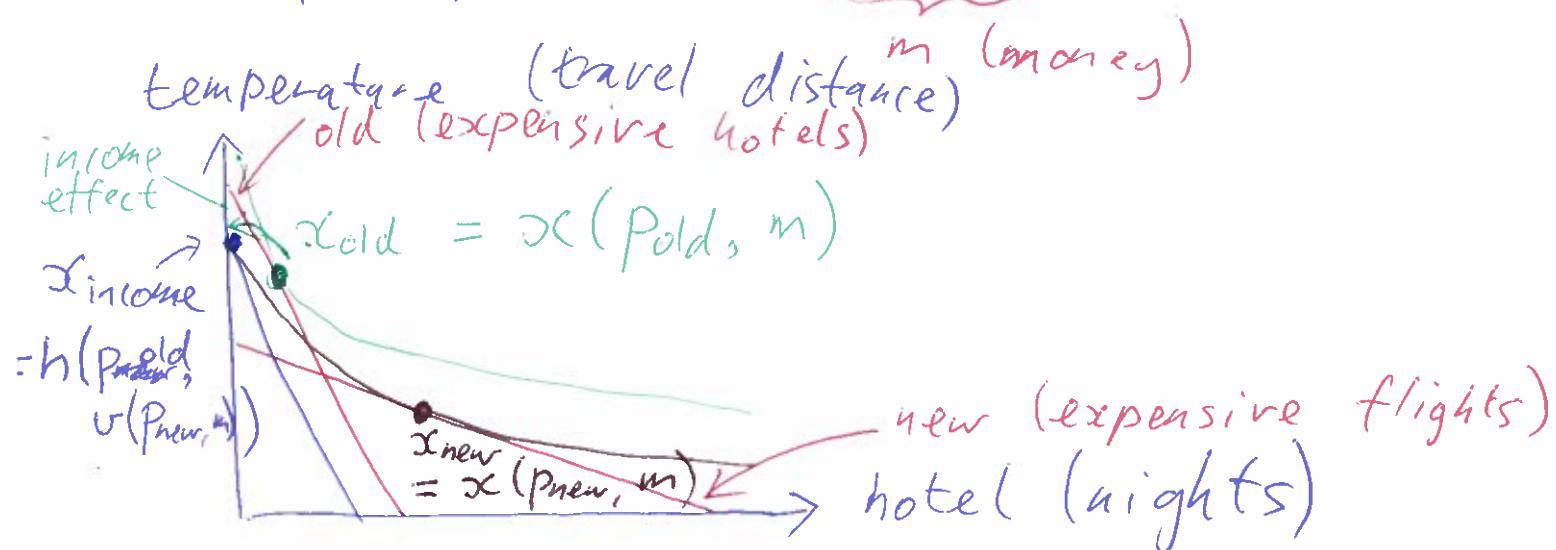


3.6 Slutsky Decomposition

In addition to the Bellman eq,
we can also connect the policy
functions:

$$h(p, \bar{u}) = x(p, \underbrace{e(p, \bar{u})}_{\text{money}}).$$



$$\text{income effect} = x^{\text{new}} - x^{\text{old}}$$

$$\text{subst'n effect} = x^{\text{new}} - x^{\text{income}}$$

$$\text{total effect} = x^{\text{new}} - x^{\text{old}}$$

Theorem 3.4 (Slutsky equation)

If $v, x(p, m), h(p, \bar{u})$ are differentiable,

then

$$\frac{\partial x_i(p, m)}{\partial p_j} = \left[\frac{\partial h_i(p, \bar{u})}{\partial p_j} \right]_{\bar{u}=v(p, m)}$$

$\underbrace{\quad}_{\text{total (net) effect}}$ $\underbrace{\quad}_{\text{subst effect}}$

$$+ \underbrace{-x_j(p, m)}_{\text{wealth loss}} \frac{\partial x_i(p, m)}{\partial m}$$

$\underbrace{\quad}_{\text{income offset}}$ $\underbrace{\quad}_{\text{officer}}$

Proof $h_i(p, \bar{u}) = x_i(p, e(p, \bar{u}))$.

Differentiate both sides w.r.t. P_j :

$$\begin{aligned}\frac{\partial h_i(p, \bar{u})}{\partial P_j} &= \left[\frac{\partial x_i(p, u)}{\partial P_j} + \frac{\partial x_i(p, u)}{\partial m} \frac{\partial e(p, \bar{u})}{\partial P_j} \right] \\ &= \left[\frac{\partial x_i(p, u)}{\partial P_j} + \frac{\partial x_i(p, u)}{\partial m} h_j(p, \bar{u}) \right]_{u=e(p, \bar{u})}\end{aligned}$$

$$\Rightarrow \frac{\partial h_i(p, \bar{u})}{\partial P_j} - \left[\frac{\partial x_i(p, u)}{\partial m} h_j(p, \bar{u}) \right]_{u=e(p, \bar{u})}$$

$$= \frac{\partial x_i(p, u)}{\partial P_j}$$

$$\Rightarrow \frac{\partial x_i(p, u)}{\partial P_j} = \left[\frac{\partial h_i(p, \bar{u})}{\partial P_j} \right]_{\bar{u}=v(p, u)} - \frac{\partial x_i(p, u)}{\partial m} x_j(p, \bar{u})$$

□

rice, beans, burgers.

Econ 1: P of rice $\uparrow \Rightarrow Q$ of beans \uparrow
(wrong!) (ignoring income effect)

Micro 1: if rice & beans are complements,
 P of rice $\uparrow \Rightarrow Q$ of beans \downarrow
 Q of burgers \uparrow .

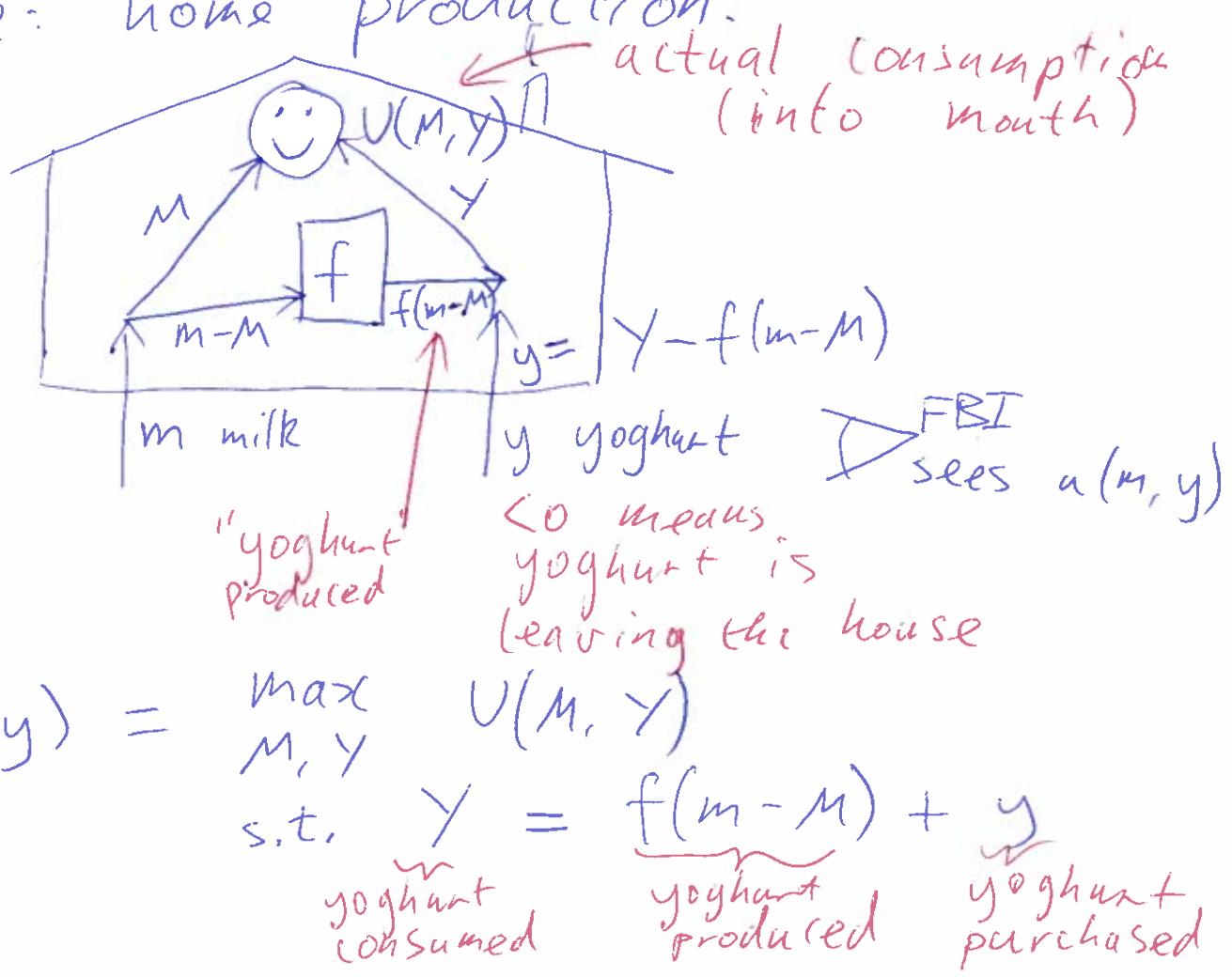
Chapter 4 - Equilibrium

4.1 Economies

Def A pure-exchange economy with N goods and H households consists of:

- * a utility function $u_h: \mathbb{R}_+^N \rightarrow \mathbb{R}$ for each household $h \in H$, and
- * an endowment $\ell_h \in \mathbb{R}_+^N$ for each household $h \in H$.

Aside: home production.



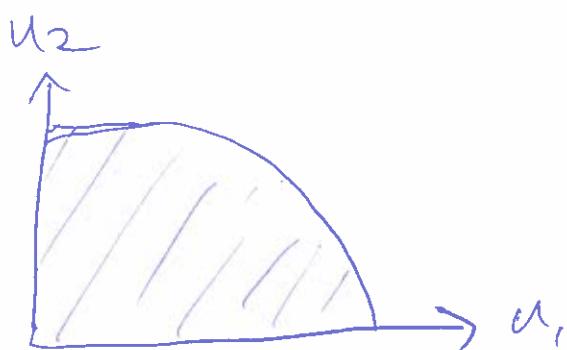
4.2 Efficient Allocations

Def The utility possibility set of an economy is the set of vectors of utilities of households for all feasible allocations. Specifically, given $(u_h, e_h)_{h \in H}$,
 $U = \{ (u_h(x_h))_{h \in H} : x \text{ is a feasible allocation} \}$

where x is feasible if

$$x_h \in \mathbb{R}_+^N \quad \text{and} \quad \sum_{h \in H} x_h = \sum_{h \in H} e_h.$$

just re-arranging
resources



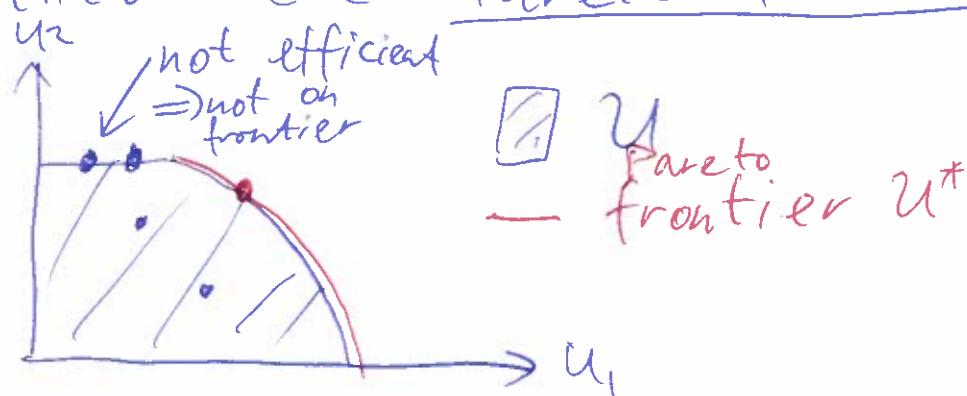
Def A vector of utilities $u \in \mathbb{R}^H$ Pareto dominates another $u' \in \mathbb{R}^H$ if

- ① no household is worse off, i.e. $u_h \geq u'_h$ for all $h \in H$, and
- ② at least one household h is strictly better off, i.e. $u_h > u'_h$ for some $h \in H$.

Def Given a utility possibility set \mathcal{U} , we say that u is ~~Pareto~~ efficient if

- ① u is feasible, i.e. $u \in \mathcal{U}$, and
- ② there is no other feasible $u' \in \mathcal{U}$ such that u' Pareto dominates u .

The set of efficient utility vectors is called the Pareto frontier, \mathcal{U}^* .



Def A social welfare function is any function $W: \mathbb{R}^H \rightarrow \mathbb{R}$.

Theorem Let $\mathcal{U} \subseteq \mathbb{R}^H$ be a utility possibility set, and $W: \mathbb{R}^H \rightarrow \mathbb{R}$ a strictly increasing welfare function. If $u \in \mathcal{U}$ maximises welfare i.e.

$$u \in \arg \max_{\hat{u} \in \mathcal{U}} W(\hat{u}),$$

then u is Pareto efficient, i.e. $u \in \mathcal{U}^*$.

4.3 Equilibrium

Def Consider a pure exchange economy (u, e) . We say that (x^*, p^*) is a pure exchange equilibrium if

$$\textcircled{1} \quad x_h^* \in \arg \max_{\substack{x_h \in \mathbb{R}_+^N}} u_h(x_h)$$

eq. allocation s.t. $p^* \cdot x_h \leq p^* \cdot e_h$, and
to household h

\textcircled{2} all markets clear, i.e. for all goods $n \in \{1, \dots, N\}$,

$$\sum_h x_{hn}^* = \sum_h e_{hn},$$

or equivalently, $\sum_h x^* = \sum_h e_h$.

4.4 Walras law & misc

Big mac ¥ 280 or £3.

Prices are relative, so we can pick any good (e.g. big macs), and set that price (of that good) to 1.

Def The excess demand function

is

$$z(p) = \sum_{h \in H} (x_h(p) - e_h) = \sum_h x_h(p) - \sum_h e_h$$

where $z: \mathbb{R}_{++}^N \rightarrow \mathbb{R}^N$.

If $z_i(p) > 0$, then at price p , there is excess demand for good i .

If $z_i(p) < 0$, then " " " excess supply"

Note: (x^*, p^*) is an equilibrium if and only if $z(p^*) = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ and $x^* = x(p^*)$.

Rewrite: ~~plus~~ ~~black leggi~~ $(x(p^*), p^*)$ is an equilibrium if and only if $z(p^*) = 0$.

Theorem 4.2 (Walras' law)

Consider a pure-exchange economy (a, e) with strictly increasing utility functions, and z be its excess demand function.

① $\underbrace{p \cdot z(p)}_{{\text{a single}} \atop {\text{number}}} = 0 \quad \text{for all } p \in \mathbb{R}_{++}^N,$ $\leftarrow p_n > 0 \text{ for all } n$

② If $N-1$ markets clear at price $p \in \mathbb{R}_{++}^N$, then all markets clear.

③ ~~$p \in \mathbb{R}_{++}^N$~~ is ~~not~~ an equilibrium price
 \Leftrightarrow there are two markets i and j such that $z_i(p) > 0$ and $z_j(p) < 0$.

Proof ① Recall household h 's budget constraint is

$$p \cdot x_h(p) = p \cdot e_h.$$

Rearrange: $p \cdot (x_h(p) - e_h) = 0.$

Sum: $\sum_h p \cdot (x_h(p) - e_h) = 0$

$$\sum_h p \cdot \sum_h (x_h(p) - e_h) = 0$$

Simpler
version of
same rule:

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

even if p is
not an equilibrium
price vector

count eq's & unknowns:

- ① FOC's HN
- ② budget N
- ③ market H
- ④ clearing

~~$HN + H + N$~~

$$\frac{x_{hn}}{p_n} \frac{HN}{N-1}$$

~~(F.O.C.)~~

$$HN + H + N - 1$$

② Without loss of generality,
suppose markets $1, \dots, N-1$ are clear,
at price $p \in \mathbb{R}_{++}^N$. Then $z_j(p) = 0$ for
 $j \in \{1, \dots, N-1\}$. Adding up,

$$\sum_{j=1}^{N-1} p_j z_j(p) = 0.$$

Next, $p \cdot z(p) - \underbrace{\sum_{j=1}^{N-1} p_j z_j(p)}_0 = p_N z_N(p) = 0.$
by ① $\Rightarrow z_N(p) = 0.$

- ③ Pick a price $p \in \mathbb{R}_{++}^N$.
- a) If there is excess supply in any market, then $z_i(p) < 0$, so $z(p) \neq 0$. If there is excess supply or demand, then ~~p~~ p is not an equilibrium price.
- b) Suppose $z(p) \neq 0$, i.e. p does not give an equilibrium. Suppose for the sake of contradiction that there is excess demand but no excess supply (similar logic needs to be checked for the opposite situation). Then $z_i(p) > 0$ for some i , and $z_j(p) \geq 0$ for all j . In this case $p \cdot z(p) > 0$, contradicting ①.