B.3 Functions

Recall that utility functions (in pure exchange economies) were written as

$$\underbrace{u_h}_{\text{name of the function}} : \underbrace{\mathbb{R}^N_+}_{\text{domain}} \rightarrow \underbrace{\mathbb{R}}_{\text{co-domain}}.$$

~~Fo~~ For every $x \in \mathbb{R}^N_+$, u_h gives an item $u_h(x)$ in the co-domain.

e.g. $f, g : \mathbb{R} \rightarrow \mathbb{R}^2$.

"f and g are functions whose domains are both $\mathbb{R}$, and co-domains are both $\mathbb{R}^2$."

Note: you can't write $\log : \mathbb{R} \rightarrow \mathbb{R}$ because log of negative numbers is undefined.

For $f : X \rightarrow Y$ to be a function, $f(x)$ needs to be well-defined for all $x \in X$.

Indicator function:

$$I(A)(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

4.5 Efficiency of Equilibria

Theorem Consider a pure-exchange economy with increasing utility functions $u_h : \mathbb{R}^N_+ \to \mathbb{R}$ and endowments ~~[illegible]~~ ~~We say that~~ $e_h \in \mathbb{R}^N_+$. If (x^*, p^*) is an equilibrium for $\underbrace{\text{this economy}}_{(u,\ e)}$, then x^* is an efficient allocation.

Proof
Suppose $\hat{x} \in \mathbb{R}^{NH}_+$ is an allocation that dominates x^*. We will show that $\hat{x}$ is infeasible. Since every dominating allocation would be infeasible, we conclude x^* is efficient.

Since each household has increasing utility, $\hat{x}_h$ can not be cheaper than x^*_h for any household h, so

$$p^* \cdot \hat{x}_h \geq p^* \cdot x^*_h.$$

Since $\hat{x}_h$ dominates x^*_h, at least one household is strictly better off — that household can't afford $\hat{x}_h$.

Adding up both sides for all households, we get

$$\underbrace{p^* \cdot \sum_{h \in H} \hat{x}_h}_{\text{market value of all consumption } \hat{x}} > \underbrace{p^* \cdot \sum_{h \in H} x^*_h}_{\text{market value of } x^* \text{ consumption}}.$$

However, all ~~xxx~~ feasible allocations have the same market value, $p^* \cdot \sum_{h \in H} e_h$. So $\hat{x}$ is infeasible. $\square$

B6 Theorems and Proofs

A theorem is a statement that has a proof (and is therefore true). Theorem that are just "stepping stones" are called "lemmas". If a theorem is an obvious consequence of another theorem, it's called a "corollary".

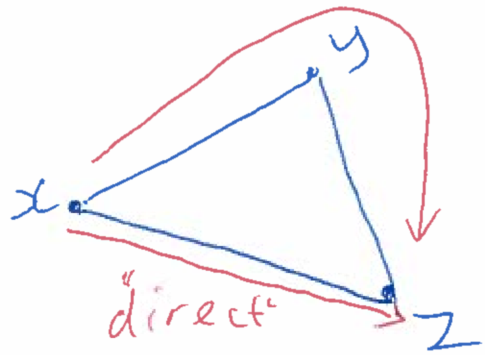
C Topology – Metric Spaces

Def (X, d) is a <u>metric space</u> where X is the "point set" and d is the "distance metric" $d: X \times X \to \mathbb{R}_+$ if

(i) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$,

(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$,

(iii) $\underbrace{d(x, z)}_{\text{distance along the direct route}} \leq \underbrace{d(x, y) + d(y, z)}_{\text{distance along the detour via } y}$ for all $x, y, z \in X$



"triangle inequality"
or
"no short-cuts"

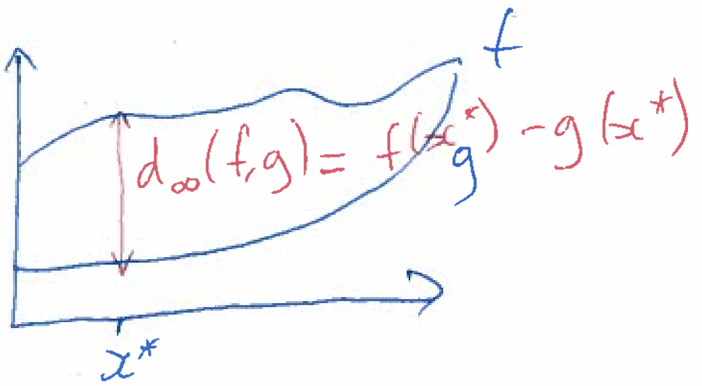
Examples

* $(\mathbb{R}^n, d_1)$ where $d_1(x, y) = \sum_{i=1}^{n} |x_i - y_i|$ "Manhattan metric"
* $(\mathbb{R}^n, d_2)$ where $d_2(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$. "Euclidean metric". (should be Pythagoras metric?)

* If (X, d) is a metric space and $Y \subseteq X$ then (Y, d) is a metric space
* $(\mathbb{R}^n, d_\infty)$ where $d_\infty(x,y) = \max_i |x_i - y_i|$
$= \max\{|x_1 - y_1|, |x_2 - y_2|, \dots, |x_n - y_n|\}$
* Function spaces. If $X = \{f : [0,1] \to [0,1]\}$ and $d_\infty(f,g) = \sup_{x \in [0,1]} |f(x) - g(x)|$, then (X, d_∞) is a metric space.

Problem with max: what is $\max [0,1)$?
Solution: supremum (sup for short)

$$\sup [0,1) = 1.$$



Things that are NOT metric spaces:
* $(\mathbb{R}^n, d)$ where $d(x,y) = \min_i |x_i - y_i|$.
* $(\mathbb{R}^n, d)$ where $d(x,y) = 0$.