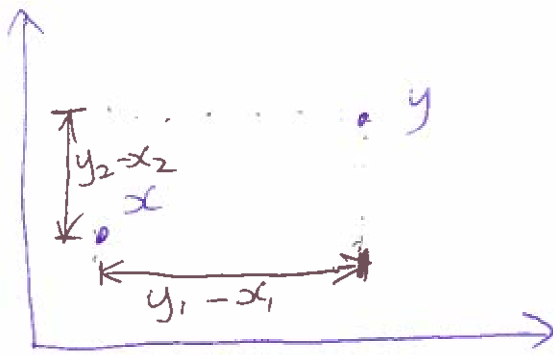
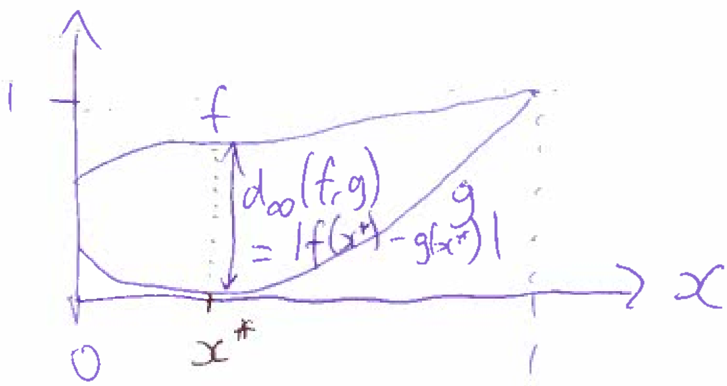
* $d_\infty(x,y) = \max_i |x_i - y_i|$



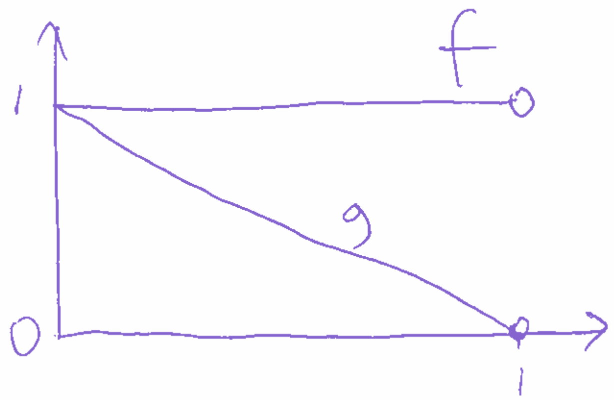
* $X = \{ f : [0,1] \to [0,1] \}$ is the set of functions whose domain and co-domain is $[0,1]$.

$d_\infty(f,g) = d_{\sup}(f,g) = \sup_x |f(x) - g(x)|$, where sup A is like the maximum, but accommodates sets like $[0,1)$. — called the <u>supremum</u> of A.

$\sup [0,1) = 1$



* $X = \{ f : [0,1) \to [0,1] \}$



$d_\infty(f,g) = 1$

No $\max_{x \in [0,1)} |f(x) - g(x)|$

* Let (X, d_X) and (Y, d_Y) be metric spaces. Let

$$\underbrace{B(X,Y)}_{\text{bounded functions from } X \text{ to } Y} = \{ f : X \to Y \text{ s.t. there is some } r > 0 \text{ s.t. for all } x, x' \in X,\ d_Y(f(x), f(x')) < r \}.$$

Then $(B(X,Y), d_\infty)$ is a metric space. Notation: $B(X) = B(X, \mathbb{R})$.

<u>Not metric spaces:</u>

* $(\mathbb{R}^n, d)$ where $d(x,y) = \min_i |x_i - y_i|$.

$d(x,y) = 0$ but $x \neq y$.

* $(\mathbb{R}^n, d)$ where $d(x,y) = 0$. Fails for the same reason.

C.2 Convergence

A <u>sequence</u> in the set X is any function with domain $\mathbb{N}$ and co-domain X. Sequences can be denoted

$$x_0, x_1, x_2, \ldots$$

$$\{x_n\}$$

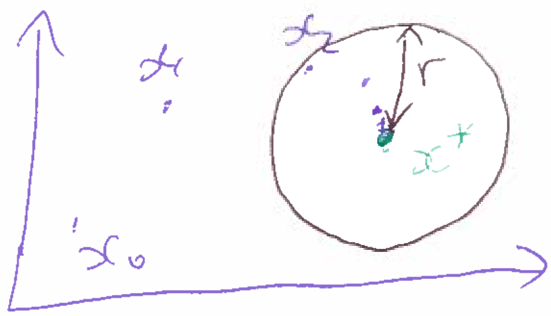
$$x_n$$

$$\{x_n\}_{n=0}^{\infty}.$$

<u>Def</u> Suppose x_n is a sequence in the metric space (X, d). We say x_n <u>converges</u> to x^* if $x^* \in X$ and for all $r > 0$, there exists some $N \in \mathbb{N}$ such that

$$d(x_n, x^*) < r \quad \text{for every } n \geq N.$$

If x_n converges to x^*, we write $x_n \to x^*$.

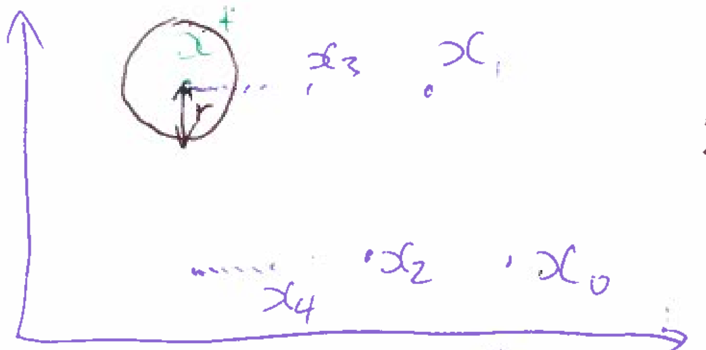


$x_n \to x^*$

Def The <u>open ball</u> of radius $r > 0$ centred at x in the metric space (X, d) is

$$B_r(x) = \{y \in X : d(x,y) < r\}.$$

Instead of writing "$d(x_n, x^*) < r$", we could write "$x_n \in B_r(x^*)$".

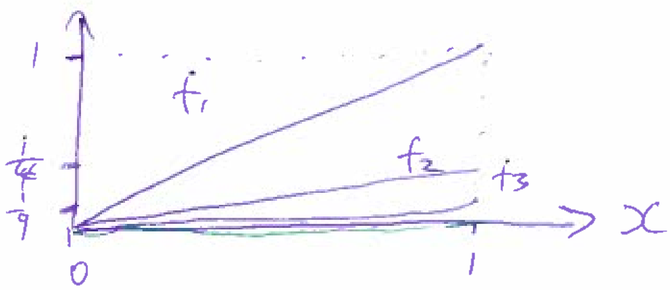


$$x_n \not\to x^*.$$

* $x_k = \frac{1}{k}$ in $(\mathbb{R}, d_2)$ converges to 0.

  (Please ignore x_0, which doesn't exist!)

* $x_k = \frac{1}{k}$ in $(\mathbb{R}_{++}, d_2)$ does <u>not</u> converge.
* $f_n(x) = \frac{x}{n^2}$ in $(B([0,1]), d_\infty)$



$f_*(x) = 0$

$f_n \to f^*$.

2.1 Production functions

Assume there are N goods in the economy. The firm produces one good, and the other $N-1$ goods are inputs. A production function $f: \mathbb{R}_+^{N-1} \to \mathbb{R}_+$ specifies how is produced $y = f(x)$ ~~when~~ from x inputs.

Assumptions:

* Possibility of inaction: $f(0) = 0$.
* Free disposal (monotonicity): If $x \geq x'$ (i.e. $x_i \geq x_i'$ for all i), then $f(x) \geq f(x')$.
* Smoothness: f is twice continuously differentiable. Each partial derivative, $\frac{\partial f(x)}{\partial x_n}$ is the marginal productive of x_n.
* Decreasing marginal productivity: e.g. $\frac{\partial f(x)}{\partial x_1}$ ~~is~~ decreases as x_1 increases (and we hold $x_2, x_3, \ldots$ fixed).

* <u>(weakly) decreasing returns to scale</u>: for all $x \in \mathbb{R}_+^{N-1}$, and all $t > 1$,

$$\underbrace{f(tx)}_{1 \text{ big firm}} \leq \underbrace{t\, f(x)}_{t \text{ small firms}}.$$

* <u>constant returns to scale</u>: for all $x \in \mathbb{R}_+^{N-1}$ and all $t > 0$,

$$f(tx) = t\, f(x).$$