# Naive Set Theory (B)

Two equals one plus one.

Two equals 1+1.

Two equals

$1+1$.

$2 = 1+1$.

The equilibrium quantity

$$Q^* = D(P^*)$$

occurs where ...

## B.1 Sets

Listing the elements exhaustively:

$$V = \{\text{attack, retreat, surrender}\}$$

logic:

$$A = \{n : n \text{ is an even number and } n < 100\}.$$

Order (or any other feature of a description of a set) does not matter.

$\{1,2,3\} = \{3,2,1\}$.

$3 \neq \{3\}$.

<u>Tuples</u> are ordered lists of items,

e.g. $(1,2,3) \neq (3,2,1)$.

$\{(milk, 3), (cheese, 2)\}$

↖ set of tuples

* $\emptyset = \{\}$ empty set
* $\mathbb{N}$ whole numbers $= \{0, 1, 2, 3, \dots\}$
* $\mathbb{Z}$ integers $= \{\dots, -2, -1, 0, 1, 2, \dots\}$
* $\mathbb{Q}$ rational numbers $= \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}, q \neq 0\}$
* $\mathbb{R}$ real numbers
* $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$

  $\mathbb{R}_{++} = \{x \in \mathbb{R} : x > 0\}$
* $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$

  $(a, b) = \{x \in \mathbb{R} : a < x < b\}$

  $(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$.

$A \subseteq B$ ("$A$ is a subset of $B$")
if for all $x \in A$, $x \in B$.

$A = B$ if $A \subseteq B$ and $B \subseteq A$.

$A \cup B$ ("the union of $A$ and $B$")
$= \{x : x \in A \text{ or } x \in B\}$.

$A \cap B$ – intersection
$= \{x : x \in A, x \in B\}$.

$A \setminus B$ – complement
$= \{x \in A : x \notin B\}$.

$A \times B$ Cartesian product
e.g. $A = \{\text{choc pudding, tart tatin}\}$
$B = \mathbb{N}$

$(\text{choc pudding}, 2) \in A \times B$

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

$$A^2 = A \times A$$

$$A^n = \underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$$

# Definitions

What does "the" mean?

Possible problems:

1. Let $x = \sqrt{-1}$. But there is no $\sqrt{-1}$! $x$ does <u>not exist</u>.

   "Let $x$ be $\boxed{\text{the}}$ square root of $-1$."

2. Let $x$ be $\boxed{\text{the}}$ square root of 4. But $\sqrt{4} = \pm 2$ — there are two of them! $x$ is <u>not unique</u>.

If the definition of $x$ suffers neither problem, we say $x$ is <u>well-defined</u>.

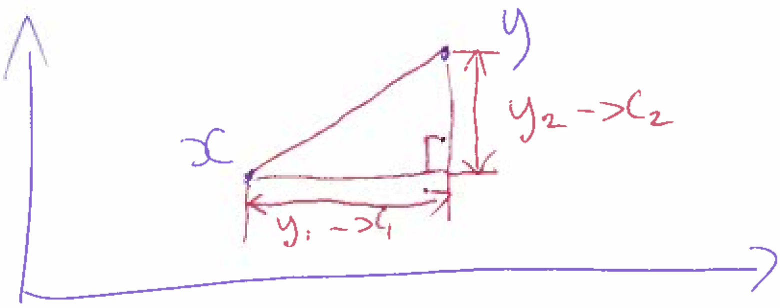
# Topology / Metric Spaces

How should we measure distance?

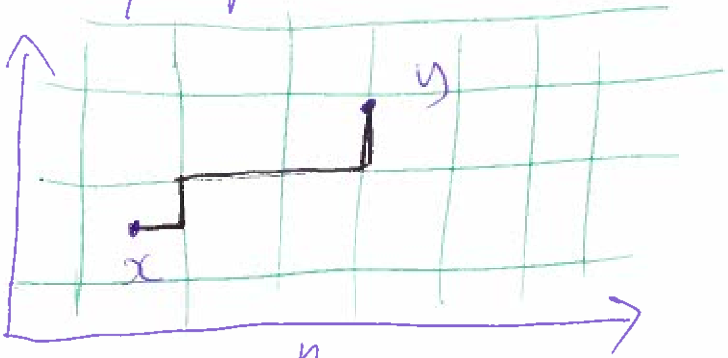
One way: $d_2(x,y)$ where $x, y \in \mathbb{R}^n$, would be to define

$$d_2(x,y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}.$$

— Pythagoras. (or Euclid)




Another proposal – 'Manhattan'




$$d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|$$

**Def** $(X, d)$ is a metric space if $X$ is a set ("point set") and $d : X \times X \to \mathbb{R}_+$ is a distance function (or metric) that satisfies:

(i) $d(x,y) = 0$ if and only if $x = y$,

(ii) $d(x,y) = d(y,x)$ for all $x, y \in X$,

(iii) $d(x,z) \leq d(x,y) + d(y,z)$ for all $x, y, z \in X$,

"triangle inequality" or "no short-cuts"

## Detour B.5 Quantifiers

"for all" vs "there exists"

"personalised" punishment

(1) For all criminals $c$, there exists a punishment $p$ such that criminal $c$ would be deterred by $p$ from crime.

(2) There exists a punishment $p$ such that for all criminals $c$, the punishment $p$ would deter criminal $c$ from crime. ← single punishment