B - Naive Set Theory

Two equals one plus one.

Two equals 1 + 1.

Two equals

$1 + 1.$

$2 = 1 + 1.$

The equilibrium quantity

Q^*

occurs where the supply and demand curves cross.

B.1 Sets

$V = \{\text{attack, retreat, surrender}\}.$

$A = \{n : n \text{ is an even number, and } n < 100\}.$

$\{1, 2, 3\} = \{\cancel{4}\, 3, 2, 1\}.$

$\{2, 3, 5, 7\} = \{n : n \text{ is a prime, } n < 10\}$

$\{\cancel{4}3\}$ singleton set.

Note: $3 \neq \{3\} \neq \{\{3\}\}$.

Christmas pudding - meal

S= {Christmas pudding, roast turkey, ...}

- Salisbury Arms Christmas Special menu

$\{\cancel{}\, S, V, H\}$ ← menu of restaurant.

H ← Howies

V ← Vittoria

(Christmas puddings, £5) — tuple

$$\underbrace{(1,2,3)}_{\text{tuple}} \neq \underbrace{(3,2,1)}_{\text{tuple}}$$

An equilibrium in Econ I consisted of $\underbrace{(P, Q)}_{\text{tuple}}$.

$(100, 4.5) \neq (4.5, 100)$

$(P, Q) \quad\quad (P, Q)$

$a \in A$ shorthand for "a is an element of A".

$\emptyset = \{\}$ empty set

$\mathbb{N} = \{0, 1, 2, \ldots\}$ natural numbers

$\mathbb{R}$ = real numbers

$\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$ – "positive real numbers"

$\mathbb{R}_{++} = \{x \in \mathbb{R} : x > 0\}$

Cartesian product:

$A \times B = \{(a, b) : a \in A, b \in B\}$.

$\{1, 2\} \times \{x, y\} = \{(1, x), (1, y), (2, x), (2, y)\}$.

$A \times A = A^2$.

B 2 Definitions

"The"

Let x be <u>the</u> number $1+1$.

Let $x = \sqrt{4}$. * $x=2$ or $x=-2$?

↑ bad, ambiguous.

Let $x = \sqrt{-1}$

↑ bad, does not exist!

If we have neither problem, x is **well-defined**.

4.1 Pure exchange economies

Def 4.1: A <u>pure exchange economy</u> with N goods and household set H consists of

$\{\underbrace{(0,0,\dots,0)}_{N}, (1,1,\dots,1)\}$

* utility function $u_h : \mathbb{R}^N_+ \to \mathbb{R}$ for each household $h \in H$, and
* endowment $e_h \in \mathbb{R}^N_+$ for each household $h \in H$.

4.2 Efficiency

Def 4.3 The utility possibility frontier of a pure exchange economy is

$$\mathcal{U} = \Big\{ (u_h(x_h))_{h \in H} \overset{\text{such that}}{:} \; x_h \in \mathbb{R}^N_+ \text{ for all } h \in H \text{ and } \underbrace{\sum_{h \in H} x_h}_{\text{demand}} = \underbrace{\sum_{h \in H} e_h}_{\text{supply}} \Big\}.$$

indexing the items in the vector

For example, $N=2$ (specifically, choc pudding, Christmas pudding), $H = \{\text{Andrew}, \text{Lucy}\}$

$$u_{\text{Andrew}}(C, X) = \cancel{\log} \sqrt{X}$$

$$u_{\text{Lucy}}(C, X) = \cancel{\log} \sqrt{C}$$

$$e_{\text{Andrew}} = (1, 1)$$

$$e_{\text{Lucy}} = (0, 1).$$

$$\mathcal{U} = \Big\{ \left(\cancel{\log} \sqrt{X_{\text{Andrew}}}, \; \cancel{\log} \sqrt{C_{\text{Lucy}}} \right) : X_{\text{Andrew}} + X_{\text{Lucy}} = 2, \; C_{\text{Andrew}} + C_{\text{Lucy}} = 1 \Big\}.$$

~~4.12~~ B.4 Statements

$X \Rightarrow Y$. eg: "$x > y$ implies $x \geq y$".

implies

stronger than

If A is true, then B implies C.

converse: If A is true, then C implies B.

"If $x > y$, then $x \geq y$."

If and only if: If A is true, then $B \Leftrightarrow C$.
equivalent

B iff C

B if and only if C

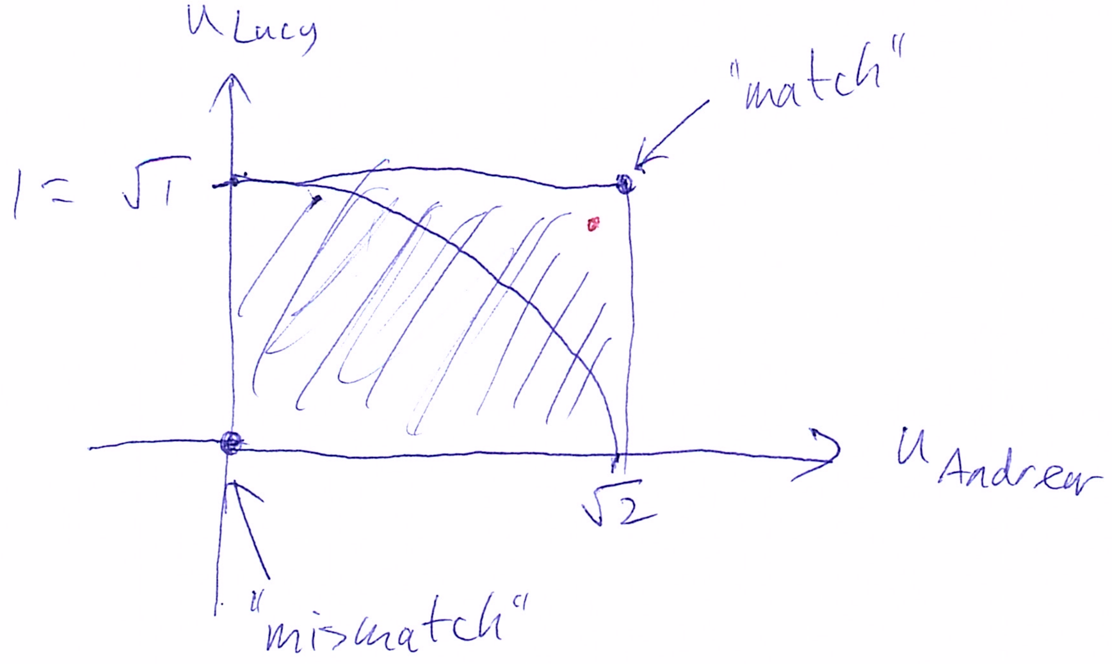
B is true if and only if C is true.

Converse of "$x > y \Rightarrow x \geq y$"

is "$x \geq y \Rightarrow x > y$" ⇐ false!

Negation: "$\underbrace{\text{not true that "} x > y \Rightarrow x \geq y \text{"}}_{\text{false}}$"

~~equivalent to~~ ~~either~~ ~~$x \leq y$~~



<u>Def</u> A vector of utilities $u \in \mathbb{R}^H$ <u>Pareto dominates</u> another vector $u' \in \mathbb{R}^H$ if

(i) no household is worse off, i.e. $u_h \geq u'_h$ for all $h \in H$, and

(ii) at least one household is strictly better off, i.e. $u_h > u'_h$ for some $h \in H$.

<u>Def</u> Given a utility possibility set $\mathcal{U}$, we say that u is <u>efficient</u> if

(i) u is feasible, i.e. $u \in \mathcal{U}$, and

(ii) there is no $u' \in \mathcal{U}$ such that u' Pareto dominates u.

original: If A, then $B \Rightarrow C$

contrapositive: If A, then "not C" $\Rightarrow$ "not B".

~~Of~~

B.5 Quantifiers

"for all" "there exists"

Compare:

(i) For all criminals c, there exists some punishment p such that c does not commit crimes

(ii) There is some punishment p such that all criminals c do not commit crimes.

Note: (i) and (ii) are different.

$$(ii) \Rightarrow (i)$$

4.3 Equilibrium

__Def__ Consider a pure-exchange economy $\{u_h\}$ and $\{e_h\}$. We say that the tuple (x^*, p^*) consisting of ~~an allocation~~ quantities x^* and prices p^* is an __equilibrium__ if:

(i) for every household $h \in H$,

$$x_h^* \in \underset{x_h \in \mathbb{R}_+^N}{\arg\max}\; u_h(x_h)$$
$$\text{s.t. } \underbrace{p^* \cdot x_h}_{\text{expenditure}} \leq \underbrace{p^* \cdot e_h}_{\text{revenue}}, \text{ and}$$

(ii) ~~for all n ∈ N~~

$$\underbrace{\sum_h x_h^*}_{\text{demand}} = \underbrace{\sum_h e_h}_{\text{supply}}.$$