Appendix H

Sample Solutions

2.6 The company buys wool and dye at prices (p_w, p_i). It allocates (w_d, w_s) units of wool to dresses and suits respectively. Similarly, it allocates (i_d, i_s) units of dye to dresses and suits respectively. This results in $f(w_d, i_d)$ and $g(w_s, i_s)$ dresses and suits being produced, which are sold at prices p_d and p_s respectively. The firm's profit maximisation problem is

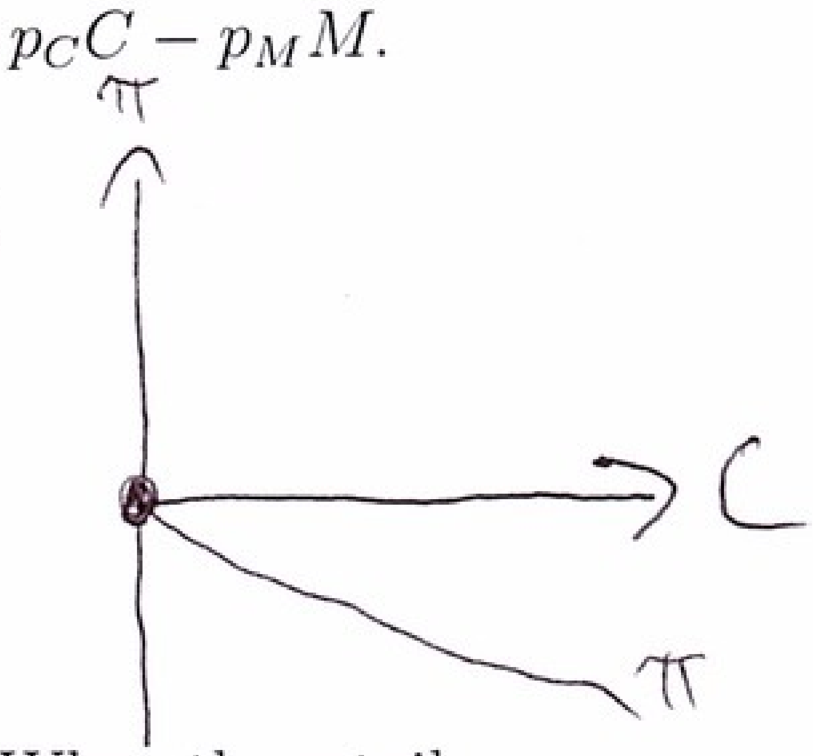
$$\pi(p_d, p_s; p_w, p_i) = \max_{w_d, w_s, i_d, i_s} p_d f(w_d, i_d) + p_s g(w_s, i_s) - p_w(w_d + w_s) - p_i(i_d + i_s).$$

2.7 Let (p_C, p_M) be the wholesale prices of chocolate and milk, and let (p_c, p_m) be the corresponding retail prices. The firm buys (C, M) units of wholesale milk and chocolate, and hires (l_c, l_m) units of labour at wage w to chocolate and milk sales, respectively. Based on these inputs, the firm sells $c(C, M, l_c, l_m)$ units of chocolate and $m(C, M, l_c, l_m)$ units of milk. (Here, we are accommodating the idea that "overselling" chocolate might reduce milk sales.) The firm's profit function is

$$\begin{aligned}
&\pi(p_c, p_m; p_C, p_M, w) \\
&= \max_{C, M, l_c, l_m \geq 0} p_c c(C, M, l_c, l_m) + p_m m(C, M, l_c, l_m) - w(l_c + l_m) - p_C C - p_M M.
\end{aligned}$$

The first-order conditions are:

$$\begin{aligned}
p_c c_C(C, M, l_c, l_m) + p_m m_C(C, M, l_c, l_m) &= p_C \\
p_c c_M(C, M, l_c, l_m) + p_m m_C(C, M, l_c, l_m) &= p_M \\
p_c c_{l_c}(C, M, l_c, l_m) + p_m m_{l_c}(C, M, l_c, l_m) &= w \\
p_c c_{l_m}(C, M, l_c, l_m) + p_m m_{l_m}(C, M, l_c, l_m) &= w.
\end{aligned}$$



These first-order conditions are only relevant for interior solutions. When the retail price is below the wholesale price, the optimal solution, $(C, M, l_c, l_m) = (0, 0, 0, 0)$ is on the boundary.

- $a_n = x$ for all $n > N$.

Therefore, $x \in A$. We conclude that $\partial A \subseteq A$ and so A is a closed set.

C.17 Consider the metric space $(\mathbb{R}, d_2)$ and the sets, $A_n = [0, 1 - 1/n]$. The union of all these sets is $[0, 1)$, which is not closed.

C.20 $N_2(1) = [0, 3)$ is an open set in $([0, 10], d_2)$, but not in $(\mathbb{R}, d_2)$.

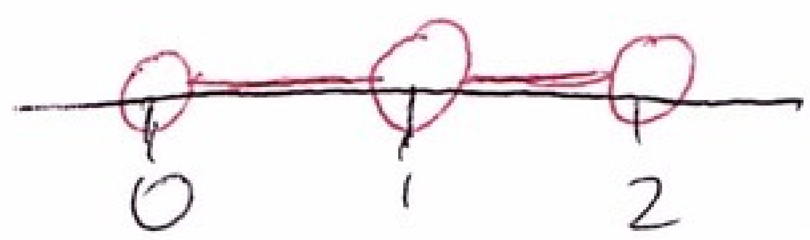
C.21 $\text{int}(A) = \{x \in \mathbb{R}^N_+ : p \cdot x < m\}$.

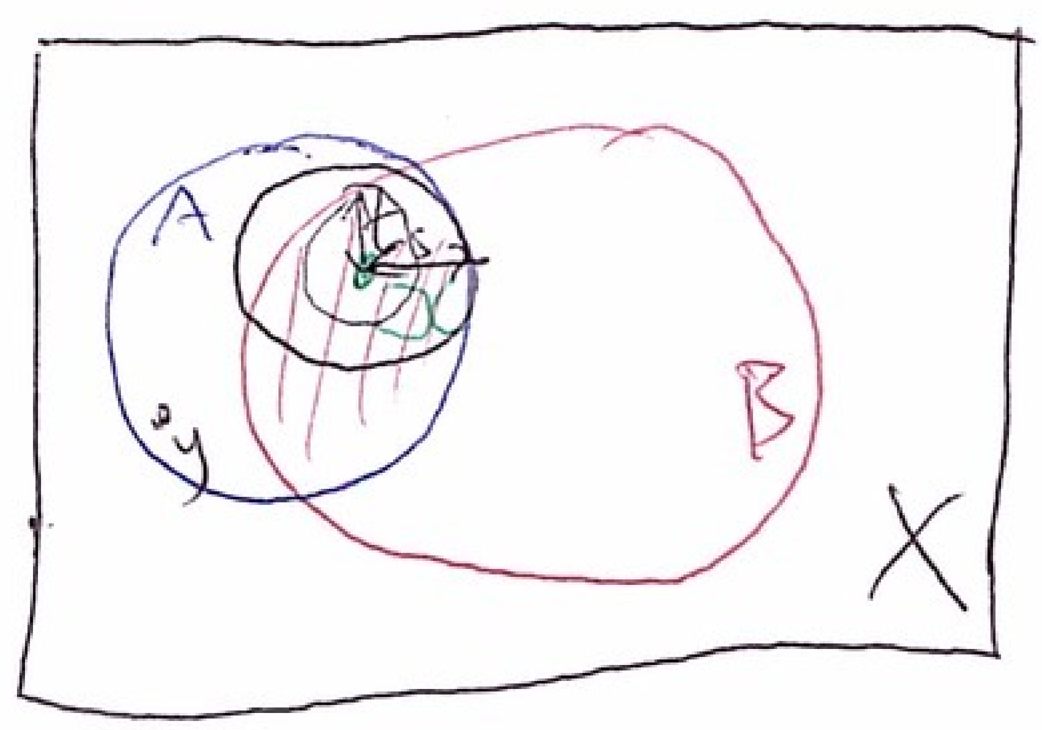
C.23 Suppose $x \in A \cap B$. We need to show that there exists some radius $r > 0$ such that $N_r(x) \subseteq A \cap B$.

Since A is open, there is some s such that $N_s(x) \subseteq A$. Since B is open, there is some t such that $N_t(x) \subseteq B$. Let $r = \min_{s,t}$. This choice implies $N_r(x) \subseteq N_s(x) \subseteq A$ and $N_r(x) \subseteq N_t(x) \subseteq B$. We conclude that $N_r(x) \subseteq A \cap B$, as required.

C.27 Consider the metric space $(\mathbb{R}, d_2)$ and the open set $A = (0, 1) \cup (1, 2)$. Then $\text{cl}(A) = [0, 2]$, and $\text{int}(\text{cl}(A)) = (0, 2) \neq A$.

$-A$





$A \cap B$

Q1 is open

$N_2(1)$ in $([0,10), d_2)$ is $[0,3)$

$\boxed{\text{C.20}}$ not open

$N_2(1)$ in $(\mathbb{R}, d_2)$ is $(-1,3)$.

Why is $[0,3)$ open in $([0,10), d_2)$?

X

Intuitive answer:

You might worry about 0 being a boundary point.

<u>Not</u> a boundary point: there is no sequence $b_n \in X \setminus [0,3)$ s.t. $b_n \to 0$.

Actual proof idea:

$s = 2 - d(1, y) = 1.5$

e.g. $y = \frac{1}{2}$

$N_{1.5}(0.5) = [0,2) \subseteq [0,3)$

No

Date

Tutorial 4 .

2.6 2.7 C.20, 21, 23, 27

2.6 ~~let x be the wool input, d = f(x) be the dresses and suits produced from x units of wool and dye, y = g(d) be the dresses and suits output from~~

let w be the wool input, d be the dye input, $a = f(w,d)$ be the dresses produced from w and d units of wool and dye respectively, $b = g(w,d)$ be the suits produced from w and d units of wool and dye respectively, P_w be the price of wool, ~~and~~ P_d be the price of dye, p_a be the price of a dress and p_b be the price of a suit. The fashion company's profit maximisation problem is :

which one?

$$\cancel{\max_{w,d} P_a \cdot a + \max_{w,d} P_b \cdot b} - P_w \cdot W - P_d \cdot d$$

$$\max_{w,d} \left[P_a f(w,d) - P_w w - P_d d \right] + \max_{w,d} \left[P_b\, g(w,d) - P_w w - P_d d \right] \cancel{- P_a a - P_b b}$$

2.7 let c be chocolate ~~bars~~ quantity, m be milk quantity, l be cashier labour input, w be cashier wage, P_c be convenience store price less wholesale price of chocolate and P_m be convenience store price less wholesale price of milk. The profit maximisation problem is:

$$\max_{c} P_c + \max_{m} P_m - lw.$$

?

No

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C20 ~~$N_2(1)$ is $[0,3)$ in the metric space $(X,d)=([0,10],d_2)$~~

$N_2(1)$ is $(-1,3)$. ✗

$N_2(1)$ is not an open set in $(X,d)=([0,10],d_2)$ because $N_2(1) \not\subseteq X$. ✗

$N_2(1)$ is an open set in $(\mathbb{R},d_2)$ because $N_2(1) \subseteq \mathbb{R}$ and does not contain its boundary.

C21. ~~Since an open set does not contain any of its boundary, the interior of the budget constraint, int(A), does not contain any of its boundary~~

~~The boundary interior of the set A is $A \cap \partial A = \emptyset$~~

$\partial A = \{x \in \mathbb{R}^N_+ : p \cdot x = m\}$ as shown previously.

$int(A) = \{x \in A \mid$ there exist $r > 0 : N_r(x) \subset A\}$
= the union of all $N_r(x)$

$\Rightarrow$ ~~In this case~~, $int(A)$ does not contain ∂A since boundary points do not have an open ball.

$\therefore int(A) = \{x \in \mathbb{R}^N_+ : p \cdot x < m\}$. ✓

C23. Since A is an open set, ~~then~~ for all $x \in A$ there exist an open ball $N_r(x)$ with $r > 0$ centred at x such that $N_r(x) \subset A$.

$\Rightarrow Int(A) = \bigcup_{x \in int(A)} N_r(x)$

This is also true for open set B. good start

$\therefore A \cap B$ which contains only the elements in A and B is also an open set ~~[illegible]~~ ?

Need $A \subseteq X$

(27) let $A = (0,1) \cup (1,2)$ in any metric space (X, d).

$cl(A) = [0,2]$

the interior of $cl(A) = (0,2) \neq A$ $\square$

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2.6 Let x be the wool input, y be the dye input, $f(x,y)$ be the dresses output, $g(x,y)$ be the suits output, p_f be the price of dresses, p_g be the price of suits, w_x be the price of wool, and w_y be the price of dye. The fashion company's profit maximization problem is:

$$\max_{x,y} p_f f(x,y) + p_g g(x,y) - w_x x - w_y y$$

used twice!

2.7 Let x_1 be the chocolate input and p_1 to be its price, x_2 be the milk input and p_2 to be its price, $f(x_1,x_5)$ be the chocolate output and p_f to be its price, $g(x_2,x_5)$ be the milk output and p_g to be its price, x_5 be the number of cashiers employed and w to be its wage. The convenience store's maximisation problem is:

$$\max_{x_1,x_2,x_5} p_f f(x_1,x_5) + p_g g(x_2,x_5) - p_1 x_1 - p_2 x_2 - w x_5$$

The first order conditions are

$$p_f \frac{\partial f(x_1^*,x_5^*)}{\partial x_1} = p_1$$

$$p_g \frac{\partial g(x_2^*,x_5^*)}{\partial x_2} = p_2$$

$p_f \frac{\partial f(x_i^*, x_s^*)}{\partial x_s} + p_g \frac{\partial g(x_i^*, x_s^*)}{\partial x_s} = w$

where the asterisks denote the value of inputs that satisfies the first order conditions

The FOC plays no role if retail prices are lower than wholesale prices. The solution will be on the boundary.

(20) $N_2(1) = \{y \in X : d(1,y) < 2\} = [0,3)$

This set is an open set in (X, d_2) but not in $(\mathbb{R}, d_2)$

(21) $\text{int}(A) = \{x \in \mathbb{R}^N_+ : p \cdot x < m\}$

(23) Assume for the sake of contradiction that $A \cap B$ is not an open set. Then there is a point $x \in A \cap B$ such that there isn't an $r \in \mathbb{R}_+$ that the set $N_r(x) \subseteq A \cap B$. Since if $x \in A \cap B$, then $x \in A$ and $x \in B$. ~~Therefore $x \in A$ is~~ why? not in the interior of A. Also $x \in B$ is not in the interior of B. Hence, A and B are not open sets. By contraposition, if A and B are open

sets, then $A \cap B$ is an open set $\square$

2.21 Consider the price vector $p \in \mathbb{R}^N_{++}$. What is the interior of the set

$A = \{x \in \mathbb{R}^N_+ : p \cdot x \leq m\}$ inside the metric space $(\mathbb{R}^N_+, d_2)$

$\text{int}(A) = \{x \in \mathbb{R}^N_+ : p \cdot x < m\}$ ✓

why?

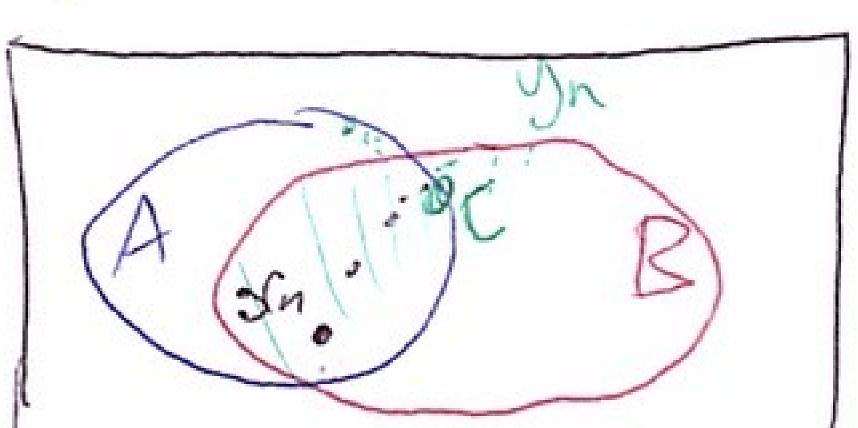
2.23 Prove that if A and B are open sets inside ~~the~~ metric space (X, d) then $A \cap B$ is an open set
let A, B be non-disjoint open sets therefore $A \cap B \subseteq A$, $A \cap B \subseteq B$. let $c \in \partial A \cap B$ then
$c \in \partial A \vee \partial B \vee \partial A \cap B \ \forall c$ but since $A \cap \partial A = B \cap \partial B = \{\emptyset\}$ $c \notin A, B \Rightarrow c \notin A \cap B \ \forall c$
therefore $\partial A \cap B \not\subseteq A \cap B$ and is an open set. For disjoint A, B, $A \cap B = \{\emptyset\}$ which is
both open and close by definition. ?

2.27 Let (X, d) be a metric space and $A \subseteq X$. Find a counter example to the following false statement
: If A is an open set then $\text{int}(\text{cl}(A)) = A$.

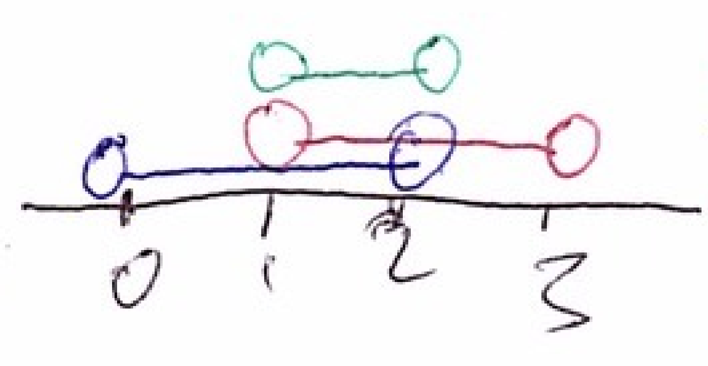
let $X = [0,1]$, $A = (0,1] \subseteq X$ then
$\text{cl}(A) = \partial A \cup A = \{0\} \cup (0,1] = [0,1] = X$
$\text{int}(\text{cl}(A)) = \text{int}(X) = [0,1] \neq A$ ✓

Suppose $c \in \partial(A \cap B)$. We want to prove that $c \notin A \cap B$. Since $c \in \partial(A \cap B)$, either $c \in \partial A$ or $c \in \partial B$. ~~...~~

Proof: Since $c \in \partial(A \cap B)$, there is a sequence $x_n \in A \cap B$ and a sequence $y_n \in X \setminus (A \cap B)$ s.t. $x_n \to c$ and $y_n \to c$.



got stuck!



$-A \cap B = (1,2)$
$-(0,2) = A$
$-(1,3) = B$

$(\mathbb{R}, d_2)$