<u>E.5</u> "Direct" proof:

Since x^* solves $\max_{x \in X} f(x)$, we know

$$f(x^*) \geq f(x) \text{ for all } x \in X.$$

Since $Y \subseteq X$, this implies

$$f(x^*) \geq f(y) \text{ for all } y \in Y.$$

could write x

So x^* solves $\max_{y \in Y} f(y)$. $\square$

<u>E.5</u> "structured" proof:

x^* solves $\max_{x \in X} f(x)$

$\Longleftrightarrow$ $f(x^*) \geq f(x)$ for all $x \in X$

"if and only if"

$\Longrightarrow$ $f(x^*) \geq f(y)$ for all $y \in Y$

"implies"

(since $Y \subseteq X$)

$\Longrightarrow$ x^* solves $\max_{y \in Y} f(y)$. $\square$

(since $x^* \in Y$)

"it has been shown"

QED

E.5 Proof by contradiction:

Suppose for the sake of contradiction that x^* does NOT solve

$$\max_{y \in Y} f(y).$$

Since Y is finite, there is some other solution $y^* \in Y$ with $y^* \neq x^*$. Since $x^* \in Y$ and x^* is not a solution, it follows that $f(y^*) > f(x^*)$. But this contradicts x^* solving

$$\max_{x \in X} f(x) \qquad \lightning$$

we found a contradiction