2.2 Profit maximisation

Assume: firms can't manipulate prices.

$N-1$ inputs
1 output (for simplicity)
$p \in \mathbb{R}_+$ output price
$w \in \mathbb{R}^{N-1}_+$ input prices (eg. wages)

$$\underbrace{\pi(p;w)}_{\text{profit function}} = \max_{\underbrace{x \in \mathbb{R}^{N-1}_+}_{\text{inputs}}} \underbrace{pf(x)}_{\text{revenue}} - \underbrace{w \cdot x}_{\text{cost}}$$

$$\cancel{= pf(x) - w}$$

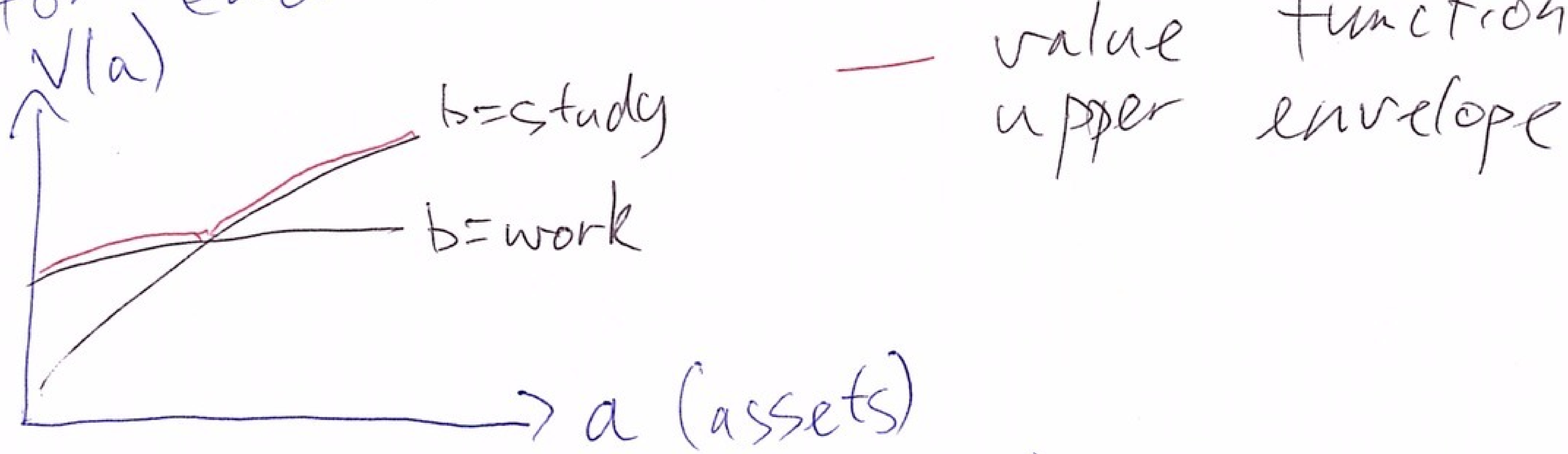
$$= p\,\underbrace{x(p;w)}_{\text{factor (input) demand functions}} - w \cdot \underline{x(p;w)}$$

First-order conditions:
If x^* is optimal, then
FOC x_i: $p \dfrac{\partial f(x^*)}{\partial x_i} = w_i$.

2.3 Upper envelopes and value functions

$$\underbrace{V(\underbrace{a}_{\text{state variable}})}_{\text{value function}} = \max_{\underbrace{b}_{\text{choice variable}}} \underbrace{v(a,b)}_{\text{objective}} = v(a, \underbrace{b(a)}_{\text{policy}})$$

V is the "upper envelope" of some functions $v(\cdot, b)$, one for each b.



Theorem (Envelope theorem)

Let $v: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ be a differentiable function, and $V(a) = \max_{b \in \mathbb{R}^m} v(a,b)$, $b(a)$ be an ~~the~~ optimal policy. If V is differentiable, then

$$V'(a) = \frac{\partial v(a,b)}{\partial a}\Big|_{b=b(a)}$$

$$= v_a(a, b(a)).$$

infinite choice set