https://andrewclausen.net

Naive set theory

Two plus two equals four.

2 +2 equals four.

Four equals

2+2.

$4 = 2+2.$

22 September 2017 09:20

The equilibrium quantity Q^* ocurs where the supply and demand curves cross. At the equilibrium quantity Q^*,

$$MC(Q^*) = MB(Q^*).$$

$V = \{\text{attack, retreat, surrender}\}$

$A = \{n : n \text{ is an even number and } n < 100\}$

$\{1, 2, 3\} = \{3, 2, 1\}$

$\{2, 3, 5, 7\} = \{n : n \text{ is a prime number and } n < 10\}$

singleton $\{3\} \neq 3$

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Tuples — order matters

$(1, 2, 3) \neq (3, 2, 1)$

(u_1, u_2)

(price, quantity) = (£10, 3000)

metric space (X, d)

2 items: pair

3 items: triple

n items: n-tuple

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$a \in A$ means "a is an element of A".

$a \notin A$ means "a is not an element of A".

eg: $1 \in \{1, 2, 3\}$

$4 \notin \{1, 2, 3\}$

$$\emptyset = \{\}$$

$$\mathbb{N} = \{0, 1, 2, \ldots\}$$ "natural numbers"

$$\mathbb{Z} = \{\ldots, -2, 1, 0, 1, 2, \ldots\}$$

"integers"

$$\mathbb{Q} = \left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}, q \neq 0\right\}$$

"rationals"

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$\mathbb{R}$ real numbers

$\mathbb{R}_+ = \{x : x \in \mathbb{R} \text{ and } x \geq 0\}.$

$R_{++} = \{x : x \in R \text{ and } x > 0\}.$

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$A \subseteq B$ means "A is a subset of B." $\subset$

If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

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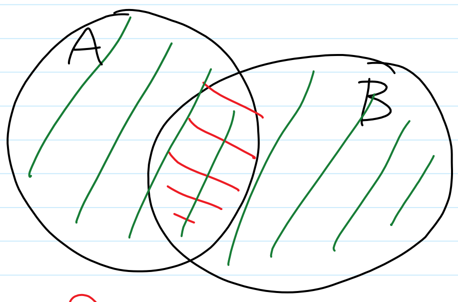
union: $A \cup B = \{x : \text{either } x \in A \text{ or } x \in B\}$

and/or

intersection: $A \cap B$.

$= \{x : x \in A \text{ and } x \in B\}$



$A \cap B$

$A \cup B$

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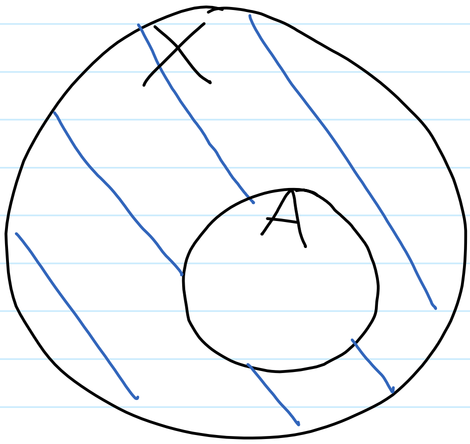
$$X \setminus A = \{x \in X : x \notin A\}$$

complement



$X \setminus A$

A and B are <u>disjoint</u> if

$A \cap B = \emptyset$.

a and a' are <u>distinct</u> if $a \neq a'$.

<u>Cartesian products</u> $A \times B$

$$= \{(a,b) : a \in A, b \in B\}.$$

e.g. $\{1,2\} \times \{x,y\} = \{(1,x),(1,y),(2,x),(2,y)\}.$

$$\emptyset \times \{1,2\} = \emptyset$$

$$\{\emptyset\} \times \{1,2\} = \{(\emptyset,1),(\emptyset,2)\}.$$

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$$A \times A = A^2$$

$$A \times A \times A = A^3$$

$$\underbrace{A \times \cdots \times A}_{n \text{ times}} = A^n$$

B.2 Functions

domain "inputs"

co-domain "possible outputs"

If f is a function with domain A and co-domain B, we write $f: A \to B$.

$f = g$?

$f(x) = x^2$

$g(x) = (\sqrt{x^2})^2$

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$$f(x,y) = xy$$

$$(x,y) \mapsto xy$$

"the function that maps (x,y) to xy"

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B3 Definitions

"let x be <u>the</u> number $1+1$."

well-defined

"Let x be <u>the</u> real number."

which one? uniqueness

"Let x be $\sqrt{-1}$."

no such thing! existence

4.1 Economies

$h \in H$ ← set of all households

<ins>Def</ins> A <ins>pure exchange economy</ins> with N goods and household set H consists of

* a utility function $u_h : \mathbb{R}_+^N \to \mathbb{R}$ for each household $h \in H$,
* an endowment $e_h \in \mathbb{R}_+^N$ for each $h \in H$.

eg: food, clothes

farmer, designer

$H = \{f, d\}$

$N = 2$

$u_F(f, c) = \sqrt{fc}$

$u_F : \mathbb{R}_+^2 \to \mathbb{R}$

$e_f = (2, 0)$

$e_d = (0, 4)$

An allocation specifies the consumption $x_h \in \mathbb{R}^N_+$ of each household $h \in H$.

Often written (x_h) or $(x_h)_{h\in H}$, or x.

x is a feasible allocation if

$$\sum_{h\in H} x_h = \sum_{h \in H} e_h \quad \leftarrow \text{vector addition}$$

$\leq$ "free disposal"

eg:

$x_f = (1, 1)$

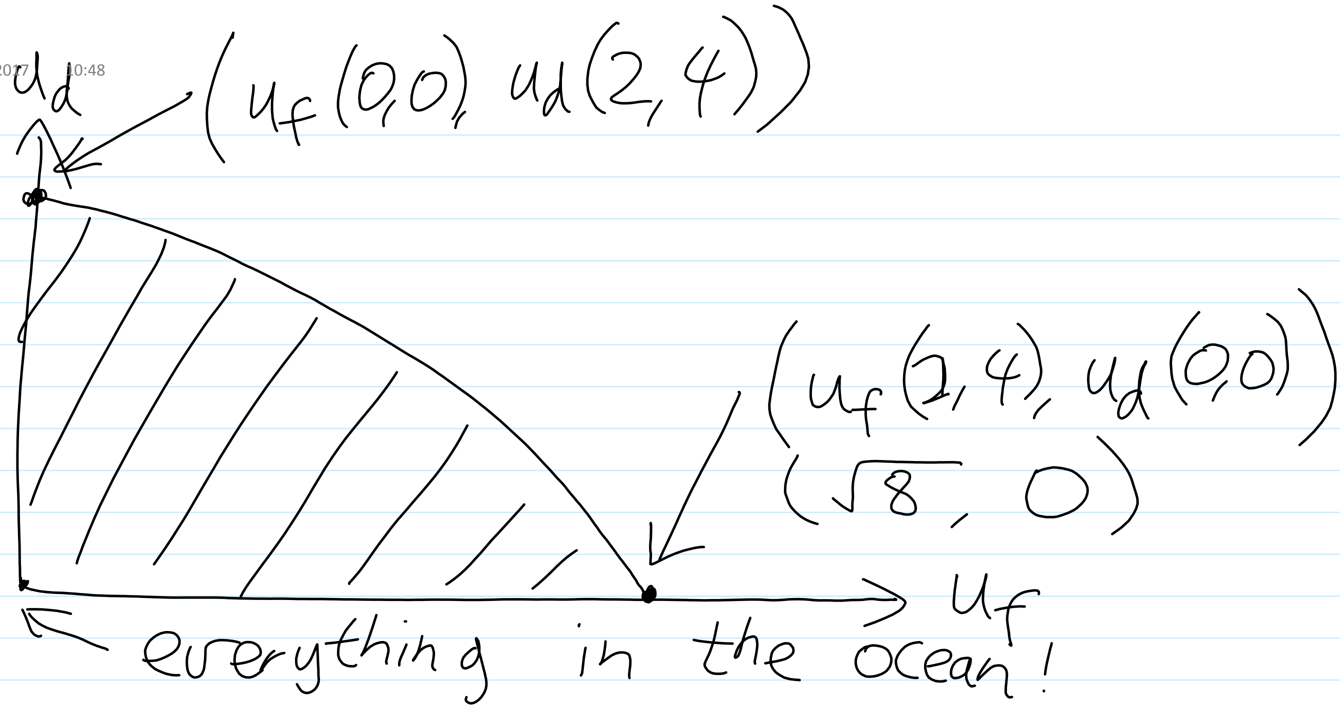
$x_d = (1, 3)$

$x_f + x_d = (2, 4)$

4.2 Efficient allocations

<u>Def</u> The <u>utility possibility set</u> of an economy

$$\mathcal{U} = \left\{ \left(u_h(x_h) \right)_{h \in H} : x \text{ is a feasible allocation} \right\}$$
$$= \left\{ \left(u_h(x_h) \right)_{h \in H} : \sum_{h \in H} x_h = \sum_{h \in H} e_h \right\}.$$


u_d
$(u_f(0,0), u_d(2,4))$
$(u_f(2,4), u_d(0,0))$
$(\sqrt{8}, 0)$
u_f
everything in the ocean!