A.64 (now (.64)

Suppose (X,d) is compact. Let $x_n \in X$ be a non-convergent sequence. Prove that there are subsequences converging to x^* and x^{**}, where $x^* \neq x^{**}$.

<u>Proof</u> Since (X,d) is compact, x_n has a convergent subsequence, $y_n \to x^*$. Since $x_n \not\to x^*$ there is some radius $r > 0$ such that for all N, there is $n > N$

such that

$$d(x_n, x^*) > r.$$

Let z_n be a subsequence of x_n such that $d(z_n, x^*) > r$ for all n.

Since (X, d) is compact, there is a convergent subsequence z_n' of z_n with $z_n' \to x^{**}$.

Clearly, $d(x^*, x^{**}) \geq r$

so $x^* \neq x^{**}$.

$\Box$

A.65 Suppose (X, d) is compact. If every convergent subsequence of $x_n \in X$ converges to the same point x^*, prove that $x_n \to x^*$.

Proof

Suppose for the sake of contradiction that x_n is non-convergent.

By A.65, x_n has subsequences

$y_n \to \bar{x}^*$ and $z_n \to \bar{x}^{**}$

with $\bar{x}^* \neq \bar{x}^{**}$.

So either $y_n \not\to x^*$ or $z_n \not\to x^*$.
This contradicts the assumption that $y_n \to x^*$ and $z_n \to x^*$.
Since x_n is convergent, and a subsequence converges to x^*, $x_n \to x^*$. $\square$

C.66 Let $A = \{ f \in B(\mathbb{R}) : f$ is increasing (weakly)$\}$.

Prove that (A, d_∞) is complete.

Proof Recall that $(B(\mathbb{R}), d_\infty)$ is complete. Just need to show that A is a closed set.

Let $f_n \in A$ with $f_n \to f^*$.
Want to show $f^* \in A$.

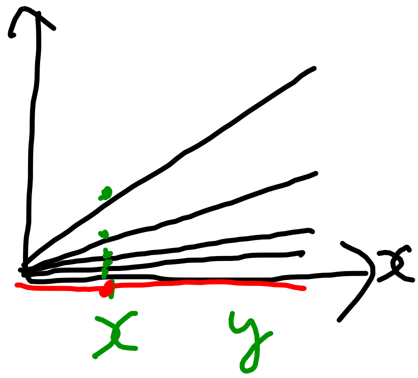
Since $f_n \in A$,

if $x \leq y$ then $f_n(x) \leq f_n(y)$.

Since $f_n \to f^*$,

$f_n(x) \to f^*(x)$

$f_n(y) \to f^*(y)$.



$\Rightarrow f^*(x) \leq f^*(y)$.

i.e. f^* is a weakly increasing function, so $f^* \in A$.

We conclude that A is closed. $\square$

2.13

(i) m^y, m^o material for artist y and o

h^y, h^o hours " "

w wage

p^m price of materials

x^y, x^o output of each artist

p^x price of paintings.

$x^y = f(m^y, h^y)$

$x^o = 2f(m^o, h^o)$.

Profit function:

$$\pi(p^x; p^m, w) = \max_{\substack{m^y, m^o \\ h^y, h^o}} p^x f(m^y, h^y) + p^x 2f(m^o, h^o) - w(h^y + h^o) - p^m(m^y + m^o).$$

(ii) Cost functions:

$$c^y(x^y; p^m, w) = \min_{m^y, h^y} p^m m^y + w h^y$$

$$\text{s.t. } 2 f(m^y, h^y) \geq x^y.$$

old

$$c^{o}(x^{o}; p^{m}, w) = c^{y}\left(\frac{x^{o}}{2}; p^{m}, w\right).$$

Bellman eq:

$$\pi(p^{x}; p^{m}, w)$$

$$= \max_{x^{y}, x^{o}} p^{x}(x^{y} + x^{o}) - c^{y}(x^{y}; p^{m}, w) - c^{y}\left(\frac{x^{o}}{2}; p^{m}, w\right).$$

(iii) $x^{y} < x^{o}$?

FOC: (of Bellman eq)

x^{y}: $\quad p^{x} = \frac{\partial c^{y}}{\partial x}(x^{y}; p^{m}, w)$

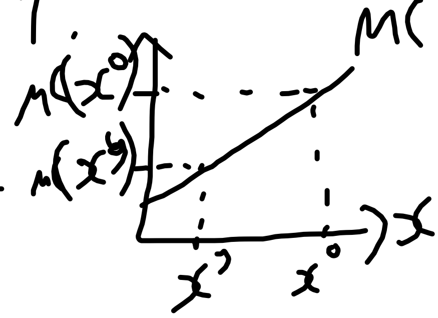
x^{o}: $\quad p^{x} = \frac{1}{2}\frac{\partial c^{y}}{\partial x}\left(\frac{x^{o}}{2}; p^{m}, w\right)$

$$\frac{\partial c^y(x^y; p^m, w)}{\partial x} = \frac{1}{2}\frac{\partial c^y(\frac{x^0}{2}; p^m, w)}{\partial x}$$

$$< 1.$$

Since f is concave, $MC \uparrow$.

$$\Rightarrow \quad x^y < \frac{x^0}{2}$$



$$\Rightarrow \quad x^y < x^0. \qquad \square$$