A.59 Prove that if U is open in (X, d), then $U \setminus \{x\}$ is also open.

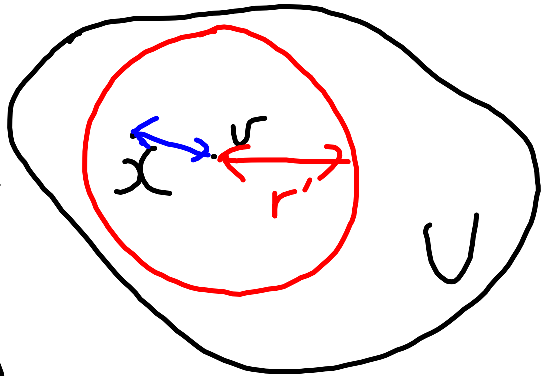
Proof Let $V = U \setminus \{x\}$.

Want to show that for every $v \in V$, there is an open ball $N_r(v) \subseteq V$.

Since $v \in U$ and U is open, there is an open ball $N_{r'}(v) \subseteq U$.

Let $r = \min\{r', d(v,x)\}$

Since $r \leq r'$,
we have $N_r(v) \subseteq U$.



Since $r \leq d(v,x)$,
we have $x \notin N_r(v)$.
We conclude that $N_r(v) \subseteq U \setminus \{x\}$
i.e. $N_r(v) \subseteq V$.
Therefore, V is an open set. $\square$

Idea: can you prove

$$\operatorname{int}(U\setminus\{x\}) = U\setminus\{x\}.$$

i.e. show

$$U\setminus\{x\} \subseteq \operatorname{int}(U\setminus\{x\})$$

A.60 Consider (X, d_1) and (X, d_2), and

$$\underbrace{f(x)=x}_{\text{identity function}}$$

$$(X, d_1) \to (X, d_2)$$
$$(X, d_1) \to (X, d_1)$$
$$(X, d_2) \to (X, d_1)$$
$$(X, d_2) \to (X, d_2).$$

Suppose f is continuous all four ways. Prove that $U \subseteq X$ is an open set in (X, d_1) iff U is open in (X, d_2).

Proof

Note that $U = f(U)$, for all sets $U \subseteq X$.

Let U be open in (X, d_1). Then since f is continuous from (X, d_2) to (X, d_1), $f^{-1}(U)$ is open in (X, d_2).

But $U = f^{-1}(U)$, so U is

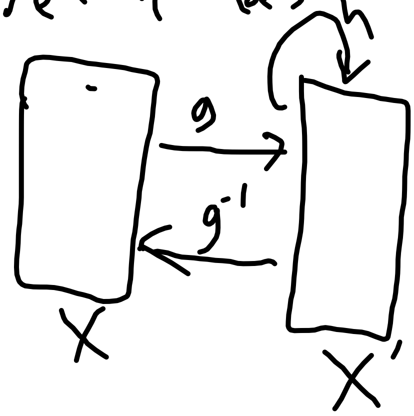
open in (X, d_2).
(Other direction is similar.) $\square$

<u>A.62</u> <u>Def</u> (X,d) has the <u>fixed point property</u> if every continuous function $f: X \to X$ has a fixed point.
Suppose (X,d) has the FPP.
Suppose (X',d') is a metric space and there is a bijection $g: X \to X'$ such that g and

g^{-1} are continuous. Prove that (X', d') has the FPP.

Proof

Let $h: X' \to X'$ be a continuous function. Want to prove: h has a fixed point.



Let $f: X \to X$ be

$$f(x) = g^{-1}(h(g(x))).$$

Since g, g^{-1}, h are continuous, f is continuous.

Since f is continuous,
f has a fixed point $x^* \in X$.
Specifically,

$$x^* = f(x^*) = g^{-1}(h(g(x^*)))$$

$$\Rightarrow g(x^*) = g(g^{-1}(h(g(x^*))))$$

$$\Rightarrow g(x^*) = h(g(x^*))$$

$$\Rightarrow y^* = h(y^*) \text{ where } y^* = g(x^*)$$

We conclude h has a fixed point. $\square$

2.10

(i) t tech (knowledge)

s silicon

l labour

w wage

p^y price of solar panels

p^s " " silicon

$y = f(t, l, s)$ solar panel output

profits:

$$\pi(\underbrace{p^y; w, p^s}_{\text{state (not chosen)}}; t) = \max_{s,l} p^y f(t,l,s) - wl - p^s s.$$

(ii) $MV(t)$?

$$\frac{\partial \pi(p^y; w, p^s; t)}{\partial t}$$

$$= \frac{\partial}{\partial t}\left[p^y f(t, l, s) - wl - p^s s\right]\Big|_{\substack{l = l(p^y; w, p^s; t) \\ s = s(\text{''})}}$$

$$= \left[p^y f_1(t, l, s)\right]_{\substack{l = l(p^y; \ldots) \\ s = s(\text{''})}}$$

$$= p^y f_1(t, l(p^y; w, p^s; t), s(\text{''})).$$

2.9 Prove firms are unresponsive to inflation.

$$\pi(tp, tw)$$

$$= \max_{x \in \mathbb{R}_+^{N-1}} tp f(x) - tw \cdot x$$

$$= t \left[\max_{x \in \mathbb{R}_+^{N-1}} p f(x) - w \cdot x \right]$$

$$= t\, \pi(p, w).$$

So $x(p, w) = x(tp, tw)$.

Firms like inflation?

No: real value of profits are unchanged.

<u>2.8</u> Consider $\pi(P,Q) = PQ - TC(Q)$.

(i) <u>Use env. theorem to calculate</u>

<u>$\frac{d}{dP}\pi(P, Q(P))$.</u>

$$V(P) = \max_{Q} \pi(P, Q)$$

$$V'(P) = \frac{\partial}{\partial P}\left[\pi(P,Q)\right]\Big|_{Q=Q(P)}$$

$$= \frac{\partial}{\partial P}\left[PQ - TC(Q)\right]\Big|_{Q=Q(P)}$$

$$= [Q]_Q = Q(p)$$

$$= Q(p).$$

(ii) What effect is ruled out?

$$V'(p) = \pi_1(p, Q(p)) + \pi_2(p, Q(p))Q'(p)$$

$$= Q(p) + \underbrace{\pi_2(p, Q(p))Q'(p)}_{=0}$$

"quantity effect"