A.4 Hint: $d'(x,y) = f(d(x,y))$.

Won't work for all f.

Which f?

A.3 Suppose (X,d) is a metric space. If

$$d'(x,y) = \min\{1, d(x,y)\},$$

prove (X,d') is a metric space.

Proof

(i) $d'(x,y) = 0$

$\iff d(x,y) = 0$

$\iff x = y$. since (X,d) is a metric space

(ii) $d'(x,y) =$

$= \min\{1, d(x,y)\}$

$= \min\{1, d(y,x)\}$

since (X,d) is a metric space

$= d'(y,x)$

(iii) $d'(x,z)$

$= \min\{1, d(x,z)\}$

$\leq \min\{1, d(x,y) + d(y,z)\}$

since (X,d) is a metric space

$\leq \min\{1, d(x,y)\} + \min\{1, d(y,z)\}$

$= d'(x,y) + d'(y,z)$. $\square$

A.7 If $d(x_n, x^*) \to 0$ then $x_n \to x^*$

$\in \mathbb{R}$ $\in X$

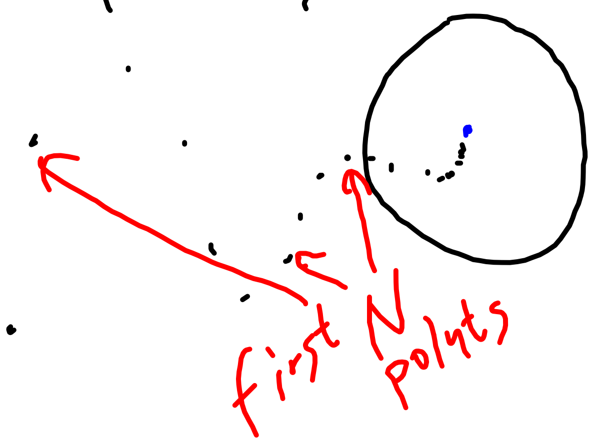
Proof

For all $r>0$, there is an N s.t. for all $n>N$,

$$d_2(0, d(x_n, x^*)) < r.$$

$$\iff d(x_n, x^*) < r$$

def. of $x_n \to x^*$. $\square$



A.8 $a_{t+1} = \frac{4}{5}(\underset{\uparrow \atop w}{20} + a_t \underset{\uparrow \atop c}{-10})$

$= \frac{4}{5}(10 + a_t)$

$= 8 + \frac{4}{5} a_t$

$= \sum_{t=0}^{\infty} \left(\frac{4}{5}\right)^t \cdot 10$

$= \frac{1}{1 - \frac{4}{5}} \cdot 10$

$= 50.$

So $a_t \to 50.$

<u>A.14</u> $A = \{ x \in \mathbb{R}^N_+ : p \cdot x \leq m \}$

Is A a closed set in $(\mathbb{R}^N_+, d_2)$?

<u>Answer</u> Yes.

$\mathbb{R}^N_+ \setminus A = \{ x \in \mathbb{R}^N_+ : p \cdot x > m \}$

$\partial A = \{ x \in \mathbb{R}^N_+ : p \cdot x = m \}$.

Since $\partial A \subseteq A$, we conclude A is closed. $\square$

A.16 If A and B are closed, then $A \cup B$ is closed.

Proof Let $x_n \in A \cup B$ be a convergent sequence with $x_n \to x^*$. We need to prove that $x^* \in A \cup B$.
There is some subsequence y_n that is entirely in A or entirely in B.
Without loss of generality, suppose $y_n \in A$.

Since A is closed, $\lim y_n \in A$.

Now since $y_n \to x^*$

we conclude

$$x^* \in A \subseteq A \cup B$$

$$\Rightarrow x^* \in A \cup B. \quad \square$$

<u>A.17</u> $[0,0] \cup \left[-\frac{1}{2}, \frac{1}{2}\right] \cup \left[-\frac{2}{3}, \frac{2}{3}\right] \cup \cdots$

$$\cancel{\left[\,[\,[\;\cdot\;]\,]\,\right]}$$

$$= (-1, 1) \leftarrow \underline{\text{NOT}} \text{ closed}$$

$$-1 + \frac{1}{n} \to -1$$