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$1+1=2.$

One plus one equals two.

Two equals $1+1$.

The equilibrium quantity

Q^*

..curs where supply = demand.

At the equilibrium quantity, Q^*,

$$MC(q^*) = MB(q^*).$$

$V = \{\text{attack, retreat, surrender}\}$.

$A = \{n : n \text{ is an even number and } n < 100\}$.

$\{1, 2, 3\} = \{3, 2, 1\}$

singleton: $\{3\} \neq 3$

tuple: $(1, 2, 3) \neq (3, 2, 1)$

(p, q)

n-tuple.

$a \in A$. "a is an element of A".

attack $\in V$.

victory $\notin V$

Special sets

$\emptyset$ empty set, $= \{\}$.

$\mathbb{N} = \{0, 1, 2, \ldots\}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Q} = \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}, q \neq 0\}$

$A \subseteq B$

$A = B$ if $A \subseteq B$ and $B \subseteq A$

$A \subset B$ if $A \subseteq B$ and $A \neq B$.

$A \cup B = \{x : x \in A \text{ or } x \in B\}$.

$A \cap B = \{x : x \in A, x \in B\}$.

A and B are <u>disjoint</u> sets if $A \cap B = \emptyset$.

$a, b \in A$ are <u>distinct</u> if $a \neq b$.

$$\underbrace{A \times B}_{\text{Cartesian product}} = \{(a,b) : a \in A, b \in B\}.$$

e.g. $\{1,2\} \times \{x,y\}$

$= \{(1,x), (2,x), (1,y), (2,y)\}$

$\emptyset \times \{1,2\} = \emptyset$

$A \times A = A^2$

$\underbrace{A \times \cdots \times A}_{n \text{ times}} = A^n.$

A function has a domain and a co-domain.

sets

$u : \mathbb{R}^n_+ \longrightarrow \mathbb{R}$

e.g. if $n=3$
$(1,2,1)$

$f : A \longrightarrow B.$

$f(x) = x^2$, $f : \mathbb{R} \rightarrow \mathbb{R}$

$g(x) = |\sqrt{x^4}|$ $f : [0,1] \rightarrow \mathbb{R}$

$g = f$ in this case.

Def 4.1 A <u>pure-exchange economy</u> with N goods and household set H consists of:

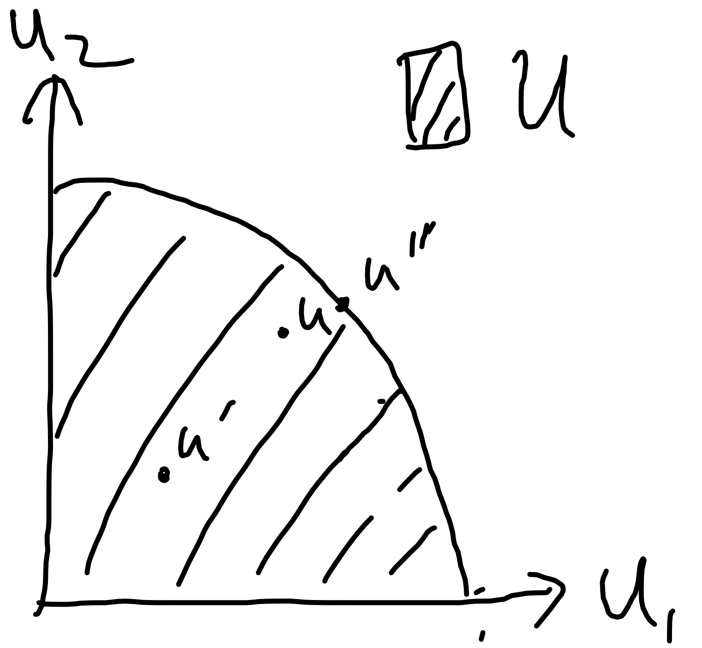
- a utility function $u_h : \mathbb{R}^N_+ \to \mathbb{R}$ for each household $h \in H$, and
- an endowment $e_h \in \mathbb{R}^N_+$ for each household.

e.g. $u(f, l) = \sqrt{fl}$.

Def 4.3 The *utility possibility set* of a pure exchange economy is

$(u_1(x_1), \dots, u_H(x_H))$

$$\mathcal{U} = \left\{ (u_h(x_h))_{h \in H} : x_h \in \mathbb{R}^N_+ \text{ for all } h \in H, \ \sum_{h \in H} x_h \leq \sum_{h \in H} e_h \right\}.$$



$(1,3)+(2,0)=(3,3)$

<u>Def 4.5</u> Given $\mathcal{U}$, we say u is <u>Pareto-efficient</u> if $u \in \mathcal{U}$ and there is no $u' \in \mathcal{U}$ that <u>Pareto dominates</u>

u, i.e. no u' such that

(i) $u'_h \geq u_h$ for all $h \in H$

(ii) $u'_h > u_h$ for some $h \in H$.

at least one

<u>Def 4.8</u> The tuple $(x^*, p^*) \in (\mathbb{R}^N_+)^H \times \mathbb{R}^N_{++}$ is a <u>pure-exchange equilibrium</u> if

(i) $x_h^* \in \underset{x_h \in \mathbb{R}_+^N}{\operatorname{argmax}} \; u_h(x_h)$

s.t. $p^* \cdot x_h$

$\leq p^* \cdot e_h$

i.e. $p^* \cdot x_h = \sum_{n=1}^{N} p_n^* x_{hn}$

and

(ii) $\underbrace{\sum_{h \in H} x_h^*}_{\text{demand}} = \underbrace{\sum_{h \in H} e_h}_{\text{supply}}$

Let $x = 1 + 1$.

Let $x = \sqrt{4}$. 2 or -2?

uniqueness?
(ambiguous def.)

Let $x = \sqrt{-1}$. $\boxed{?}$

existence problem.

We say an object is <u>well-defined</u> (can say "the") if it exists and is unique.

Statements & Quantifiers

$\cos^2 x + \sin^2 x = 1$ for all $x \in \mathbb{R}$

true or false

X implies Y or "$X \Rightarrow Y$"
if either X is false or
Y is true.

X is a stronger statement.

$x \in \mathbb{Z}$ x is an even number

X

Y

"for all" ← the rest of the statement has ~~to~~ to be true no matter which x we choose.

"there exists" — at least one.

Theorem, proof, lemma, corollary

If A, then B implies C.

<u>Converse</u>: If A, then C implies B.

(often not true)

If A, then B $\underset{\sim}{\text{iff}}$ C

if and only if

If $x>y$ then $x\neq y$.

If $x\neq y$ then $x>y$.

partial converse → or $x<y$

Theorem 4.3

Consider a pure-exchange economy with increasing utility functions

$$u_h : \mathbb{R}^N_+ \to \mathbb{R}$$

for each household and endowments $e_h \in \mathbb{R}^N_+$.
If (x^*, p^*) is an equilibrium, then x^* is Pareto efficient.

Proof: For the sake of contradiction, assume that $\hat{x}$ is feasible and Pareto dominates x^*.

Since u_h is increasing, $\hat{x}_h$ can't be cheaper than x^*_h (i.e. $p^* \cdot \hat{x}_h$ can't be $< p^* \cdot x^*_h$).

$\Rightarrow p^* \cdot \hat{x}_h \geq p^* \cdot x^*_h$

Since $\hat{x}$ Pareto dominates x^*, there exists some household $\hat{h}$ s.t.

$$p^* \cdot \hat{x}_{\hat{h}} > p^* \cdot x^*_{\hat{h}}.$$

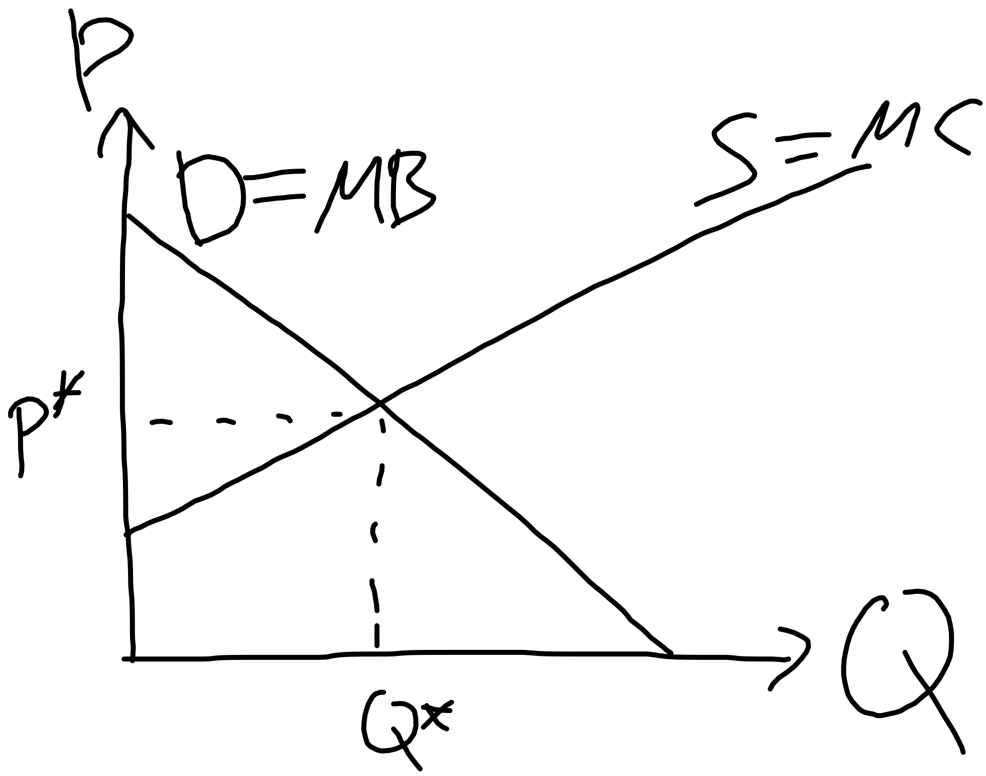
Adding up:

$$\sum_{h \in H} p^* \cdot \hat{x}_h > \sum_{h \in H} p^* \cdot x^*_h.$$

$$p^* \cdot \sum_{h \in H} \hat{x}_h > p^* \cdot \sum_{h \in H} x^*_h$$

$$= \sum_h e_h$$

↯ □


P
D=MB
S=MC
P*
Q*
Q

Contrapositive:

"$A \Rightarrow B$"

"$\neg B \Rightarrow \neg A$" } equiv.

w/ negation